

Push The Pace #4

You have thirty-five minutes to answer seven examination questions

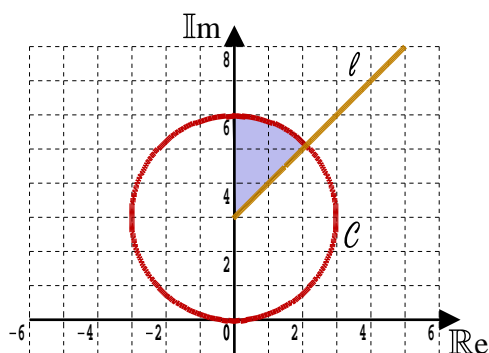
Marks Available : 40 (+ 14 bonus)

Further A-Level Pure Mathematics

Push The Pace Revision Papers

Question 1

Further A-Level Examination Question from June 2013, Paper 1, Q6 (OCR)



The Argand diagram shows a half-line ℓ and a circle \mathcal{C} .

The circle has centre $3i$ and passes through the origin.

- (i) Write down, in complex number form, the equations of ℓ and \mathcal{C} .

[4 marks]

- (ii) Write down inequalities that define the region shaded.
(The shaded region includes the boundaries)

[3 marks]

Question 2

Further A-Level Examination Question from June 2019, Paper 1, Q7 (MEI)

A curve has cartesian equation $(x^2 + y^2)^2 = 2c^2 xy$, where c is a positive constant.

(a) Show that the polar equation of the curve is $r^2 = c^2 \sin 2\theta$

[2 marks]

(b) Sketch the curves $r = c\sqrt{\sin 2\theta}$ and $r = -c\sqrt{\sin 2\theta}$ for $0 \leq \theta \leq \frac{\pi}{2}$

[3 marks]

(c) Find the area of the region enclosed by one of the loops in part (b).

[3 marks]

Question 3

Advanced Higher Examination Question from May 2019, Q2, (SQA)

Matrix **A** is defined by $\mathbf{A} = \begin{pmatrix} 2 & 1 & 4 \\ -3 & p & 2 \\ -1 & -2 & 5 \end{pmatrix}$ where $p \in \mathbb{R}$

(a) Given that the determinant of **A** is 3, find the value of p

[3 marks]

Matrix **B** is defined by $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ q & 3 \\ 4 & 0 \end{pmatrix}$ where $q \in \mathbb{R}$

(b) Find **AB**

[2 marks]

(c) Explain why **AB** does not have an inverse.

[1 mark]

Question 4

Further AS-Level Examination Question from May 2019, Q10 (WJEC)

The quadratic equation,

$$px^2 + qx + r = 0$$

has roots α and β , where p, q, r are non-zero constants.

(a) A cubic equation is formed with roots $\alpha, \beta, \alpha + \beta$

Find the cubic equation with coefficients expressed in terms of p, q, r .

[6 marks]

(b) Another quadratic equation $px^2 - qx - r = 0$ has roots 2α and γ .

Show that $\beta = -2\gamma$

[3 marks]

Question 5

Further A-Level Specimen Examination Question from 2017, Paper 1, Q11 (AQA)

(a) Prove that $\frac{\sinh \theta}{1 + \cosh \theta} + \frac{1 + \cosh \theta}{\sinh \theta} \equiv 2 \coth \theta$

[4 marks]

(b) Solve $\frac{\sinh \theta}{1 + \cosh \theta} + \frac{1 + \cosh \theta}{\sinh \theta} = 4$

Give your answer in an exact form.

[2 marks]

Question 6

Further A-Level Examination Question from June 2015, Paper FP1, Q3 (WJEC)

The complex number z satisfies the equation,

$$2z - iz^* = \frac{2 + i}{1 - i} \quad \text{where } z^* \text{ denotes the complex conjugate of } z$$

Express z in the form $a + bi$ where a and b are rational numbers to be found.

[4 marks]

Question 7

Further A-Level Examination Question from June 2010, Paper FP2, Q8 (Edexcel)

- (a) Find the value of λ for which $y = \lambda x \sin 5x$ is a particular integral of the differential equation,

$$\frac{d^2y}{dx^2} + 25y = 3 \cos 5x$$

[4 bonus marks]

- (b) Using your answer to part (a), find the general solution of the differential equation,

$$\frac{d^2y}{dx^2} + 25y = 3 \cos 5x$$

[3 Bonus Marks]

Given that at $x = 0$, $y = 0$ and $\frac{dy}{dx} = 5$,

- (c) find the particular solution of this differential equation, giving your solution in the form $y = f(x)$

[5 bonus marks]

- (d) Sketch the curve with equation $y = f(x)$ for $0 \leq x \leq \pi$

[2 bonus marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk