Push The Pace #3

You have thirty-five minutes to answer seven examination questions

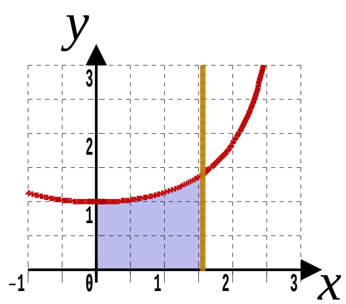
Marks Available : 40 (+ 10 bonus)

Further A-Level Pure Mathematics
Push The Pace Revision Papers

Question 1

Further A-Level Examination Question from June 2019, Paper 1, Q4 (OCR)

The graph shows the region bounded by the curve $y = sec\left(\frac{x}{2}\right)$, the x-axis, the y-axis and the line $x = \frac{\pi}{2}$



This region is rotated through 2π radians about the x-axis.
Find, in exact form, the volume of the solid of revolution generated.

[3 marks]

Question 2

Further AS-Level Examination Question from June 2016, Paper FP1. Q1 (CEA)

Let $\mathbf{A} = \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(i) Verify that $\mathbf{A}^2 = 3\mathbf{A} - 2\mathbf{I}$

[4 marks]

(ii) Hence, or otherwise, express the matrix $\mathbf{A}^{-1}$ in the form $\alpha\mathbf{A} + \beta\mathbf{I}$, where α and β are real numbers.

[3 marks]

Question 3

Further A-Level Examination Question from June 2019, Paper 1, Q8 (OCR)

The roots of the equation $x^3 - x^2 + kx - 2 = 0$ are α, $\frac{1}{\alpha}$ and β

(a) Evaluate, in exact form, the roots of the equation.

[6 marks]

(b) Find k

[2 marks]

Question 4

Further AS-Level Examination Question from June 2021, Paper 1, Q8 (AQA)

Stephen is correctly told that $(1 + \mathrm{i})$ and -1 are two roots of the polynomial equation $z^3 - 2\mathrm{i}z^2 + pz + q = 0$ where p and q are complex numbers.

(a) Stephen states that $(1 - \mathrm{i})$ **must** also be a root of the equation because roots of polynomial equations occur in conjugate pairs.
Explain why Stephen's reasoning is wrong.

[1 mark]

(b) Find p and q

[5 marks]

Question 5

Further A-Level Examination Question from June 2019, Paper 1, Q4 (AQA)

Solve the equation $2z - 5\mathrm{i}z^* = 12$

[4 marks]

Question 6

Further A-Level Examination Question from June 2015, Paper FP3, Q1 (WJEC)

(a) Express $5\,cosh\,\theta + 3\,sinh\,\theta$ in the form $r\,cosh\,(\theta + \alpha)$, $r > 0$, where the values of r and α are to be found.

[4 marks]

(b) Hence solve the equation $5\,cosh\,\theta + 3\,sinh\,\theta = 10$

[4 marks]

Question 7

Further A-Level Examination Question from June 2017, Paper FP3, Q6 (WJEC)

The integral I_n is given, for $n \geqslant 0$, by $I_n = \int_0^{\frac{\pi}{4}} tan^n\, x\, dx$

(a) Show that, for $n \geqslant 2$, $I_n = \dfrac{1}{n - 1} - I_{n-2}$

Hint : set up a chain rule backwards keeping in mind that
the derivative of $tan\, x$ is $sec^2\, x$

[2 + 3 BONUS marks]

(b) Hence determine the value of the integral, $\int_0^{\frac{\pi}{4}} \left(3 + tan^2 x \right)^2 \, dx$

leaving your answer in terms of π

[7 BONUS marks]