Push The Pace #2

You have thirty-five minutes to answer seven examination questions

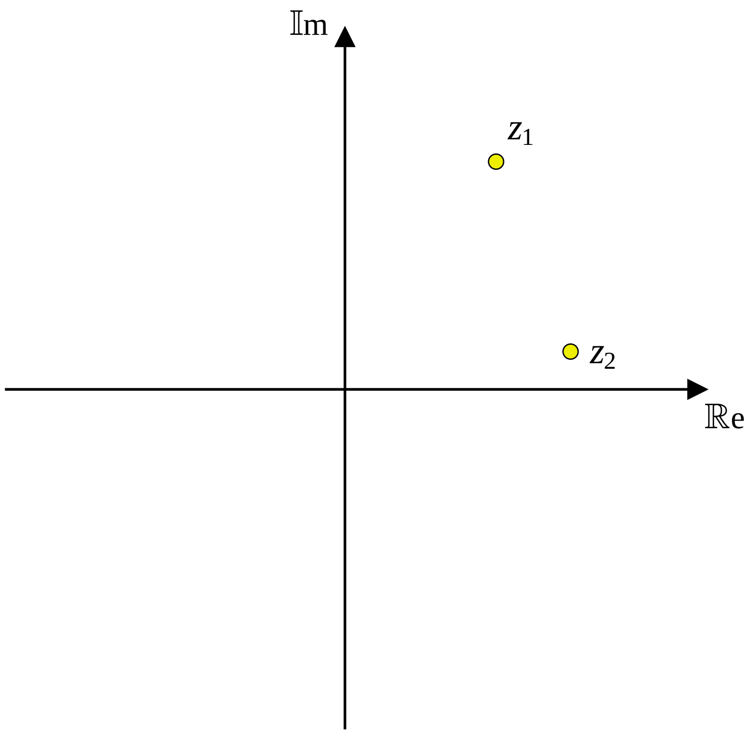
Marks Available : 40 (+ 6 bonus)

Further A-Level Pure Mathematics
Push The Pace Revision Papers

Question 1

Further AS-Level Examination Question from October 2020, Paper 1, Q2 (OCR)

The Argand diagram shows two complex numbers z_1 and z_2



(a) Mark points representing each of the following complex numbers,

- z_1^*
- $z_2 - z_1$

[2 marks]

(b) In the case where $z_1 = 1 + 2\text{i}$ and $z_2 = 3 + \text{i}$, find $\dfrac{z_2 - z_1}{z_1^*}$ in the form $a + b\text{i}$, where a and b are real numbers.

[2 marks]

Question 2

Further A-Level Examination Question from June 2022, Paper 4, Q1 (WJEC)

A function f has domain $(-\infty,\ \infty)$ and is defined by $f\,(x)\ =\ cosh^3 x\ -\ 3\,cosh\,x$

(a) Show that the graph of $y\ =\ f\,(x)$ has only one stationary point.

[4 marks]

(b) Find the nature of this stationary point.

[3 marks]

(c) State the largest possible range of $f\,(x)$

[1 mark]

Question 3

Further A-Level Examination Question from June 2019, Paper 2, Q2 (WJEC)

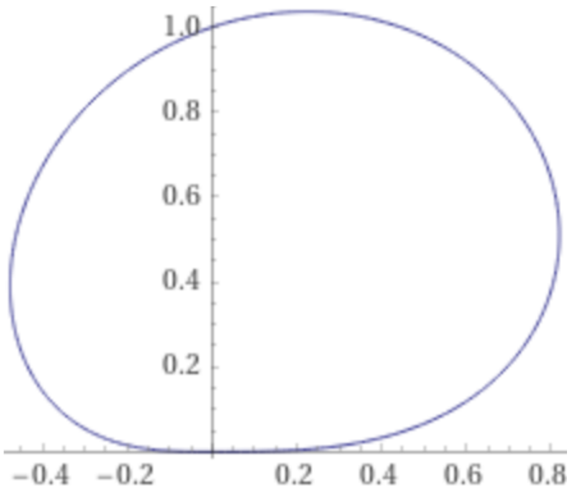
When plotted on an Argand diagram, the four fourth roots of the complex number $9 - 3\sqrt{3}\,\mathrm{i}$ lie on a circle. Find the equation of this circle.

[4 marks]

Question 4

Further A-Level Examination Question from June 2019, Paper 2, Q9 (OCR)

The diagram shows the curve $r = \sqrt{\sin\theta}\, e^{\frac{1}{3}\cos\theta}$ for $0 \leqslant \theta \leqslant \pi$



(a) Find the exact area enclosed by the curve.

[4 marks]

(b) Show that the greatest value of r on the curve is $\sqrt{\dfrac{\sqrt{3}}{2}}\ e^{\frac{1}{6}}$

[7 marks]

Question 5

Further A-Level Examination Question from June 2019, Paper 2, Q10 (OCR)

(a) Use differentiation to find the first two non-zero terms of the Maclaurin expansion of $ln\left(\frac{1}{2} + cos\,x\right)$

[4 marks]

(b) By considering the root of the equation $ln\left(\frac{1}{2} + cos\,x\right) = 0$ deduce that $\pi \approx 3\sqrt{3\,ln\left(\frac{3}{2}\right)}$

[3 marks]

Question 6

Further A-Level Examination Question from June 2020, Paper 2, Q12 (AQA)

(a) Given that $I = \int_a^b e^{2t} \sin t \, dt$, show that $I = \left[q e^{2t} \sin t + r e^{2t} \cos t \right]_a^b$

where q and r are rational numbers to be found.

[6 marks]

(b) A small object is initially at rest. The subsequent motion of the object is modelled by the differential equation,

$$\frac{dv}{dt} + v = 5e^{t} \sin t$$

where v is the velocity at time t

Find the speed of the object when $t = 2\pi$, giving your answer in exact form.

[6 BONUS marks]