<u>The mean value of a function</u>

Learning Objectives

- To be able to find the mean value of a function and apply transformations.
- To be able to apply a range of integration techniques including by substitution and by parts.

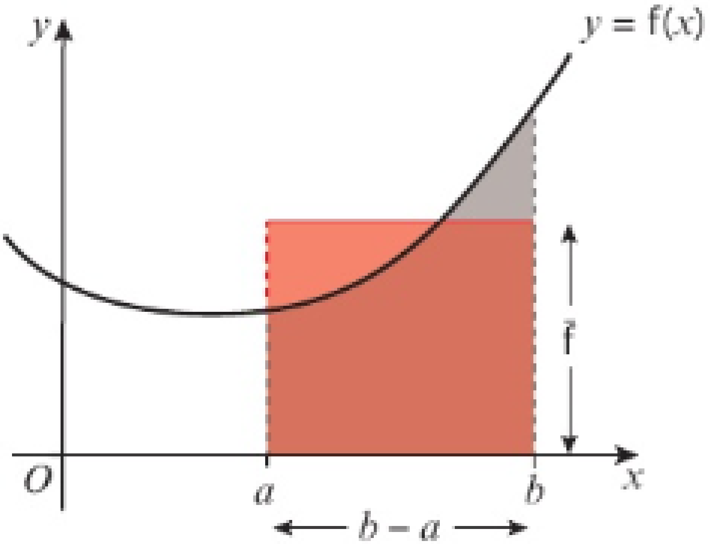
Starter Questions

1. $\int 2x^2(x^3 - 1)^3 dx$

2. $\int \frac{\sin\theta}{1+\cos 2\theta} d\theta$

3. $\int_0^{\frac{\pi}{6}} x \sin(3x)\, dx$

Mean Value Function



Example 1

Find the mean value of $f(x) = \dfrac{4}{\sqrt{2 + 3x}}$ over the interval [2, 6].

Example 2

Find the exact mean value of
$f(x) = x\cos 2x$ over the interval $\left[0, \dfrac{\pi}{2}\right]$.

Question Practice

1. Find the exact mean value of $f(x) = \dfrac{\sin x \cos x}{\cos 2x + 2}$ over the interval $\left[0, \dfrac{\pi}{2}\right]$.

2. Find the exact mean value of $f(x) = x \sin 2x$ over the interval $\left[0, \dfrac{\pi}{3}\right]$.

3.

$f(x) = \ln(kx)$, where k is a positive constant.

Given that the mean value of $f(x)$ on the interval $[0, 2]$ is -2, find the value of k. **(4 marks)**

If the function $f(x)$ has mean value $\bar{f}$ over the interval $[a, b]$, and k is a real constant, then:

- **$f(x) + k$ has mean value $\bar{f} + k$ over the interval $[a, b]$**
- **$kf(x)$ has mean value $k\bar{f}$ over the interval $[a, b]$**
- **$-f(x)$ has mean value $-\bar{f}$ over the interval $[a, b]$.**

Watch out You cannot deduce the mean value of $f(-x)$ or $f(kx)$ in this way.

4.

$f(x) = x(x^2 - 4)^4$

a Show that the mean value of $f(x)$ over the interval $[0, 2]$ is $\frac{256}{5}$ **(3 marks)**

b Use the answer to part **a** to find the mean value over the interval $[0, 2]$ of $-2f(x)$. **(2 marks)**

5.

$$f(x) = \frac{\cos x}{(2 + \sin x)^2}$$

a Find $\int f(x)\,dx$. **(4 marks)**

b Hence show that the mean value of $f(x)$ over the interval $\left[0, \frac{5\pi}{3}\right]$ is $-\frac{3}{130\pi}(3 + 4\sqrt{3})$. **(2 marks)**

c Hence, or otherwise, find the mean value, over the interval $\left[0, \frac{5\pi}{3}\right]$, of $f(x) + 3x$. **(3 marks)**

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3.

$$f(x) = \frac{1}{\sqrt{4x^2 + 9}}$$

(a) Using a substitution, that should be stated clearly, show that

$$\int f(x)dx = A\sinh^{-1}(Bx) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence find, in exact form in terms of natural logarithms, the mean value of $f(x)$ over the interval [0, 3].

(2)

NB: From the formula booklet

Integration (+ constant; $a > 0$ where relevant)

$f(x)$	$\int f(x)\,dx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$
$\frac{1}{\sqrt{a^2 - x^2}}$	$\arcsin\left(\frac{x}{a}\right)$ $(\lvert x\rvert < a)$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{x^2 - a^2}}$	$\text{arcosh}\left(\frac{x}{a}\right)$, $\ln\{x + \sqrt{x^2 - a^2}\}$ $(x > a)$
$\frac{1}{\sqrt{a^2 + x^2}}$	$\text{arsinh}\left(\frac{x}{a}\right)$, $\ln\{x + \sqrt{x^2 + a^2}\}$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a + x}{a - x}\right\rvert = \frac{1}{a}\text{artanh}\left(\frac{x}{a}\right)$ $(\lvert x\rvert < a)$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x - a}{x + a}\right\rvert$

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Reflection

Before this lesson I could…

During this lesson I learnt that…

I now need to…