Lesson 5

GCSE Mathematics
Iteration

5.1 Rational Results

Last lesson we looked at iterations that converged upon integers. However, just as the answers obtained by using algebra to solving equation can be rational numbers, so can an iteration converge upon such a number. The ideas involved are the same but it may take more iterations, more presses of the = sign, to become reasonably confident of what fraction the iteration is converging upon.

5.2 By Algebra

Use algebra to solve the equation, $x = \dfrac{(1 - x)}{2}$

[3 marks]

5.3 By Iteration

(i) With $A_1 = 1$ and $A_{n+1} = \dfrac{(1 - A_n)}{2}$ complete the table,

Term	Value
A_1	1
A_2	
A_3	
A_4	
...	...
A_{10}	
...	...
A_{20}	

[4 marks]

(ii) What does the limit of this iterative sequence seems to be ?

[1 mark]

(iii) Show that your part (ii) answer is a fixed point of the iteration.

[3 marks]

5.4 Exercise

Non-Calculator

Marks Available : 40

Question 1

(i) With $B_1 = 1$ and $B_{n+1} = \dfrac{(1 + B_n)}{4}$ complete the table,

Term	Value
B_1	1
B_2	
B_3	
B_4	
...	...
B_{10}	
...	...
B_{20}	

[4 marks]

(ii) What does the limit of this iterative sequence seems to be ?

[1 mark]

(iii) Show that your part (ii) answer is a fixed point of the iteration.

[3 marks]

Question 2

Show that $\dfrac{1}{7}$ is a fixed point of the iteration $C_{n+1} = \dfrac{1 + C_n}{8}$

[3 marks]

Question 3

(i) Complete the table to show what sevenths look like, expressed as decimals.

Vulgar Fraction	Decimal Fraction
$\frac{1}{7}$	0.1428571429...
$\frac{2}{7}$	0.2857142857...
$\frac{3}{7}$	
$\frac{4}{7}$	
$\frac{5}{7}$	
$\frac{6}{7}$	

[2 marks]

(ii) With $D_1 = 1$ and $D_{n+1} = \frac{(3 + D_n)}{8}$ complete the table,

Term	Value
D_1	1
D_2	
D_3	
D_4	
...	...
D_{10}	

[4 marks]

(iii) What rational number does the limit of this iterative sequence seems to be ?

[1 mark]

(iv) Show that your part (ii) answer is a fixed point of the iteration.

[3 marks]

Question 4

(i) With $E_1 = 1$ and $E_{n+1} = \dfrac{(5 - 4E_n)}{3}$ complete the table,

Term	Value
E_1	1
E_2	
E_3	
E_4	
...	...
E_{10}	
...	...
E_{20}	
...	...
E_{40}	

[4 marks]

(ii) This iteration is “not convergent”.
Explain what this means.

[1 mark]

(iii) Show that $\dfrac{5}{7}$ is a fixed point of the iteration
(that the iteration failed to find)

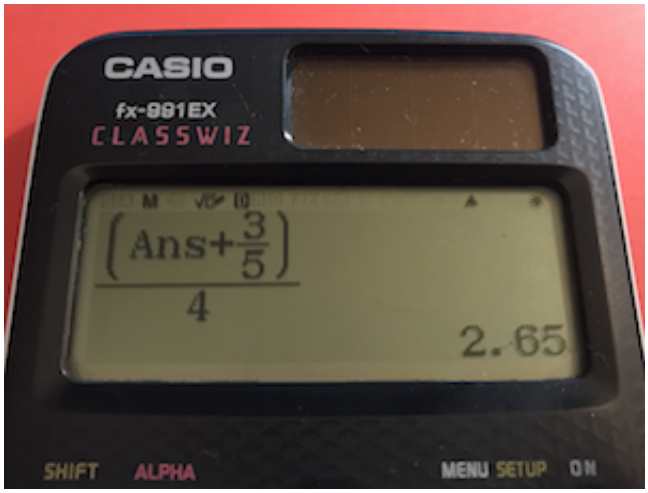
[3 marks]

Question 5

The equation $x = \dfrac{\left(x + \frac{3}{5}\right)}{4}$ is to be solved using iteration.

(i) Use $F_1 = 10$ and type in $F_{n+1} = \dfrac{\left(F_n + \frac{3}{5}\right)}{4}$ into your calculator.

It should look something like this,



[1 mark]

(ii) With $F_1 = 10$ and $F_{n+1} = \dfrac{\left(F_n + \frac{3}{5}\right)}{4}$ complete the table,

Term	Value
F_1	10
F_2	
F_3	
F_4	
...	...
F_{10}	
...	...
F_{20}	

[4 marks]

(iii) What rational number does the limit of this iteration seems to be ?

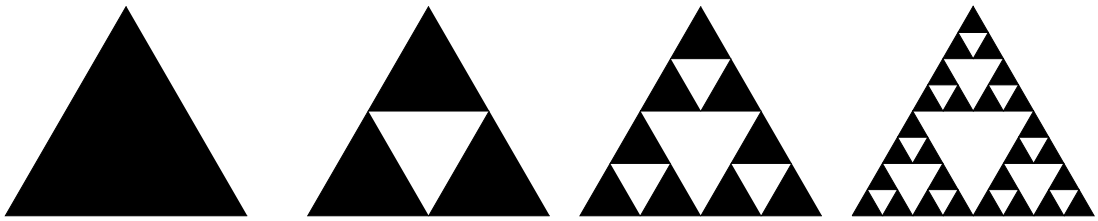
[1 mark]

(iv) Show that your part (iii) answer is a fixed point of the iteration.

[4 marks]

(v) What conclusion can you reach regarding $x = \dfrac{\left(x + \frac{3}{5}\right)}{4}$?

[1 mark]



If you don't understand iteration, just try it a few times

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