Lesson 4

GCSE Mathematics
Iteration

4.1 "Doing The Same to Both Sides"

Use the algebra of "doing the same to both sides" to solve $x = \frac{x + 20}{5}$

[3 marks]

4.2 Solving Using Iteration

The equation $x = \frac{x + 20}{5}$ is to be solved using iteration.

(i) With $A_1 = 10$ and $A_{n+1} = \frac{A_n + 20}{5}$ complete the following table,

Term	Value
A_1	10
A_2	
A_3	
A_4	
A_5	
A_6	
A_7	
A_8	

[4 marks]

(ii) What does the limit of this iterative sequence seems to be ?

[1 mark]

(iii) Show that your part (ii) answer is a fixed point of the iteration.

[1 mark]

(iv) Complete your answer by writing a conclusion.

[1 mark]

4.3 Exercise

You may use a calculator

Marks Available : 50

Question 1

The equation $x = \frac{x + 6}{2}$ is to be solved using iteration.

(i) With $B_1 = 10$ and $B_{n+1} = \frac{B_n + 6}{2}$ complete the following table,

Term	Value
B_1	10
B_2	
B_3	
B_4	
B_5	
B_6	
B_7	
B_8	

[4 marks]

(ii) What does the limit of this iterative sequence seems to be ?

[1 mark]

(iii) Show that your part (ii) answer is a fixed point of the iteration.

[1 mark]

(iv) Complete your answer by writing a conclusion.

[1 mark]

Question 2

GCSE Examination Question from November 2018, Paper 1MA1/3H Q13 (Edexcel)

The number of animals in a population at the start of year t is P_t

The number of animals at the start of year 1 is 400

Given that $P_{t+1} = 1.1\, P_t$ work out the number of animals at the start of year 3

[2 marks]

Question 3

The equation $x = 6 - \frac{8}{x}$ is to be solved using iteration.

(i) With $C_1 = 10$ and $C_{n+1} = 6 - \frac{8}{C_n}$ complete the following table,

Term	Value
C_1	10
C_2	
C_3	
C_4	
C_5	
C_6	
C_7	
C_8	

[4 marks]

(ii) What does the limit of this iterative sequence seems to be ?

[1 mark]

(iii) Show that your part (ii) answer is a fixed point of the iteration.

[1 mark]

(iv) Complete your answer by writing a conclusion.

[1 mark]

(v) This iteration has another positive integer fixed point less than ten.
Try to guess what this might be.

Check your guess by rerunning the iteration with C_1 equal to your guess.

If $C_2 = C_1$ your guess is correct !

[3 marks]

Question 4

The equation $x = \frac{18}{x} - 7$ is to be solved using iteration.

(i) With $D_1 = 10$ and $D_{n+1} = \frac{18}{D_n} - 7$ complete the following table,

Term	Value
D_1	10
D_2	
D_3	
D_4	
D_5	
D_6	
D_7	
D_8	

[4 marks]

(ii) What does the limit of this iterative sequence seems to be ?

[1 mark]

(iii) Show that your part (ii) answer is a fixed point of the iteration.

[1 mark]

(iv) Complete your answer by writing a conclusion.

[1 mark]

(v) This iteration has a positive integer fixed point less than ten.
Try to guess what this might be.

Check your guess by rerunning the iteration with D_1 equal to your guess.

If $D_2 = D_1$ your guess is correct !

[3 marks]

Question 5

The equation $x = -\left(\frac{15}{x} + 8\right)$ is to be solved using iteration.

(i) With $E_1 = 10$ and $E_{n+1} = -\left(\frac{15}{E_n} + 8\right)$ complete the following table,

Term	Value
E_1	10
E_2	
E_3	
E_4	
E_5	
E_6	
E_7	
E_8	

[4 marks]

(ii) What does the limit of this iterative sequence seems to be ?

[1 mark]

(iii) Show that your part (ii) answer is a fixed point of the iteration.

[1 mark]

(iv) Complete your answer by writing a conclusion.

[1 mark]

(v) This iteration has another negative integer fixed point close by.
Try to guess what this might be.

Check your guess by rerunning the iteration with E_1 equal to your guess.

If $E_2 = E_1$ your guess is correct !

[3 marks]

Question 6

The equation $x = \frac{9}{x^2} - \frac{3}{x} - 5$ is to be solved using iteration. This question is about investigating the associated iteration, $F_{n+1} = \frac{9}{(F_n)^2} - \frac{3}{F_n} - 5$

(i) Show that $F_1 = 1$ is a fixed point of the iteration.

[2 marks]

(ii) Show that $F_1 = -3$ is another fixed point of the iteration.

[2 marks]

(iii) This iteration is very slow to get to it's fixed point.
Show this by completing the following table.

Term	Value
F_1	10
F_2	
...	...
F_{10}	
...	...
F_{50}	
...	...
F_{100}	

[7 marks]

Mathematician's would say that this iteration is “slow to converge”.