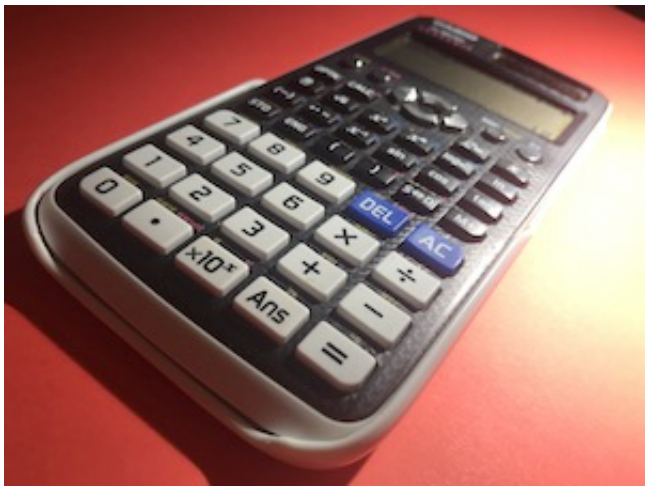
Lesson 3

GCSE Mathematics
Methods of Solving Equations

3.1 The Button Marked "Ans"

Photograph by Martin Hansen



- Program your calculator's answer buttor with the number 2 by tping in;
- 2 =
- Now type in;
- 2 × Ans = = = = = = = =

This cunning use of the calculator generates a sequence of numbers starting from the 3. Write down the sequence in the following table,

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
2							

[4 marks]

There are two ways of describing the sequence generated.

3.2 The Position-to-Term Rule

This takes the position, n, and turns it into the term in that position.
For our example the position-to-term rule is,

$$A_n = 2^n$$

So if you want to know what the 6th term is in the sequence you take that 6 and work out "2 to the power 6".

$$A_6 = 64$$

Question

Find the value of A_{10} using the position-to-term rule.

[2 marks]

3.3 The Term-to-Term Rule

This is an iterative description of the sequence and is more in tune with how we generated the series using the Ans button. We specify the starting term and then the rule for getting from one term to the next.

For our example the term-to-term rule is,

$$A_1 = 2, \quad A_{n+1} = 2A_n$$

Question

Find the value of A_{10} using the term-to-term rule.

[2 marks]

3.4 Example

A sequence has a term-to-term description of,

$$B_1 = 1, \quad B_{n+1} = \frac{1}{2}B_n + 1$$

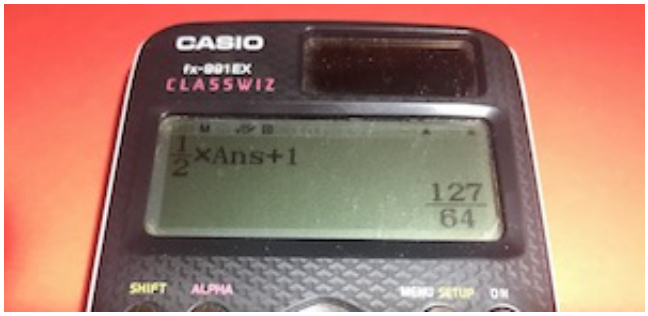
(i) Use your calculator to find the first eight terms of the sequence.
Write the term in the following table and in vulgar fraction form,

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8
1							

[4 marks]

(ii) Find a position to term formula for the sequence.

[4 marks]



Photograph by Martin Hansen

3.5 Exercise

You may use a calculator

Marks Available : 50

Question 1

- Program your calculator's answer buttor with the number 3 by typing in;
- 3 =
- Now type in;
- 3 × Ans = = = = = = = =

This cunning use of the calculator generates a sequence of numbers starting from the 3.

(i) Write down the sequence in the following table,

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
3							

[4 marks]

(ii) Write down the position-to-term rule for the sequence.

[2 marks]

(iii) Write down the term-to-term rule for the sequence.

[2 marks]

Question 2

The term-to-term formula for a sequence, D, is,

$$D_1 = 1 \qquad D_{n+1} = \left(\sqrt{D_n} + 1\right)^2$$

(i) Complete the table to show the first eight terms of the sequence.

D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8

[4 marks]

(ii) Write down the position-to-term formula for these numbers.

[2 marks]

(iii) What is the special name given to these numbers ?

[1 mark]

Question 3

(i) Prof Loo Pin has given you the following term-to-term rule to investigate.

$$P_1 = 1 \qquad P_{n+1} = 3 - P_n$$

Complete the table to show the first six terms of the sequence.

P_1	P_2	P_3	P_4	P_5	P_6

[4 marks]

(ii) What will be the value of P_{100} ?

[2 marks]

(iii) Prof Loo Pin now wants you to investigate,

$$Q_1 = 3, \quad Q_{n+1} = 3 - Q_n$$

Complete the table to show the first six terms of the sequence.

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6

[4 marks]

(iv) What will be the value of Q_{1000} ?

[2 marks]

(v) Prof Loo Pin now wants you to investigate,

$$R_1 = \frac{1}{2}, \quad R_{n+1} = 3 - R_n$$

Complete the table to show the first six terms of the sequence.

R_1	R_2	R_3	R_4	R_5	R_6

[4 marks]

(vi) What will be the value of $R_{1000000}$?

[2 marks]

(vii) Prove that, no matter what number you start with, sequences of this type will always behave in the manner you have observed in the previous parts of this question.

[4 marks]

Question 4

A number sequence, U, has the following iterative description,

$$U_1 = 1 \qquad U_{n+1} = \frac{2}{3} U_n + 1$$

(i) Complete the table to show the first eight terms of the sequence.

U_1	U_2	U_3	U_4	U_5	U_6

[4 marks]

(ii) The position-to-term rule for the sequence is $U_n = \dfrac{3^n - 2^n}{3^{n-1}}$

Show that this correctly works out U_6

[2 marks]

Question 5

GCSE Examination Question from November 2021 Paper 1MA1/2H Q16(a) (Edexcel)

Use the iteration formula $x_{n+1} = \sqrt[3]{10 - 2x_n}$ to find the values x_1, x_2 and x_3

giving your answers to 9 decimal places.

Start with $x_0 = 2$

[3 marks]

Question 6

GCSE Examination Question from November 2019, Paper 1MA1/2H Q22 (Edexcel)

The number of rabbits on a farm at the end of month n is P_n

The number of rabbits at the end of the next month is given by $P_{n+1} = 1.2\,P_n - 50$

At the end of March there are 200 rabbits on the farm.

(a) Work out how many rabbits there will be on the farm at the end of June.

[3 marks]

(b) Considering your results in part (a), suggest what will happen to the number of rabbits on the farm after a long time.

[1 mark]