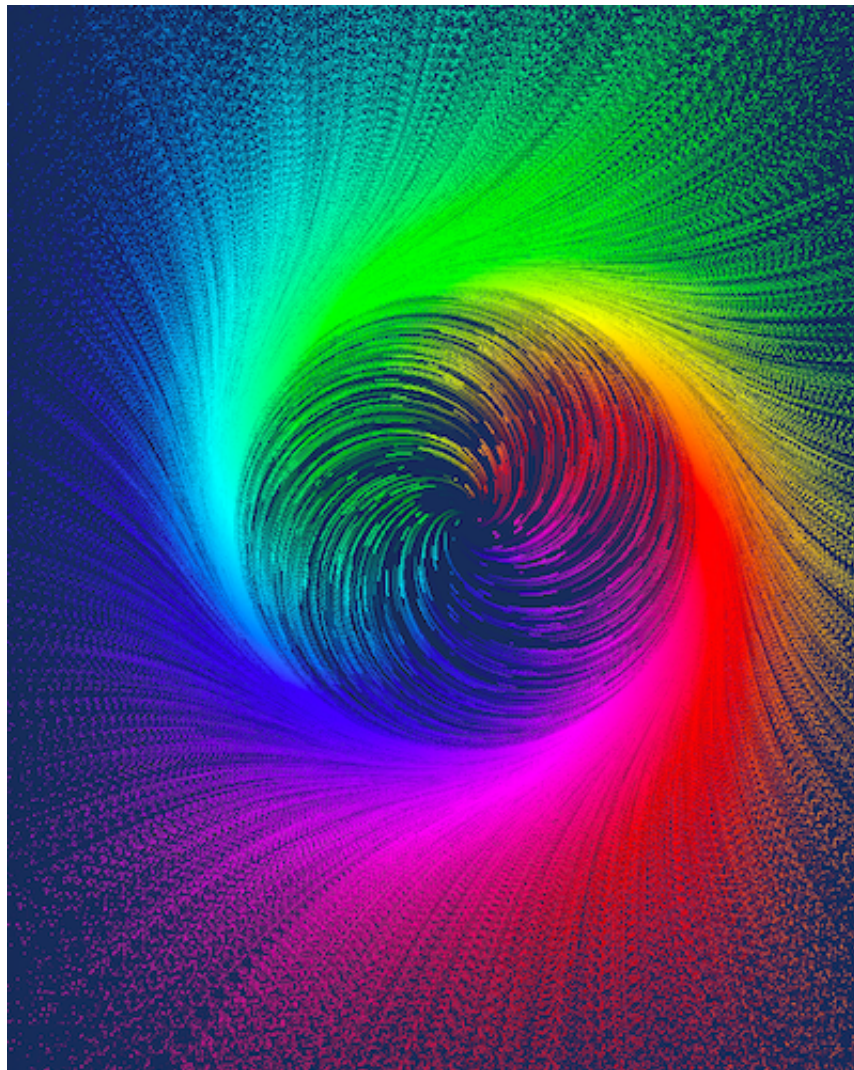


GCSE Mathematics

METHODS of SOLVING EQUATIONS



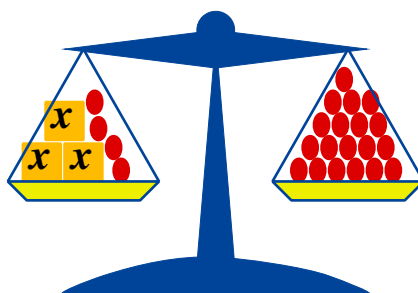
METHODS of SOLVING EQUATIONS

Lesson 1

GCSE Mathematics Methods of Solving Equations

1.1 The Preferred Method

Solving any equation with a single unknown in it, usually called x , is about finding the value of x that makes the equation true (there may be more than one). The preferred method is to make use of the idea that an equation is like a set of scales and that, so long as “the same is done to both sides” the essence of the equation is preserved.



Write down the equation is associated with the balance pictured and solve it using the algebra of doing the same to both sides.

[3 marks]

1.2 More Complicated Equations

As the equations to be solved become more complicated the approach using purely algebra, although still preferred, becomes increasingly difficult to push through to an answer.

Suppose we wish to solve the equation, $x^3 + 2x - 33 = 0$

In the grand scheme of things, this equation is not particularly complicated and yet it is not at all easy to solve by doing the same to both sides.

Rather than pack up our bags and go home, we need a “Plan B” method of solving equations for when the preferred method is difficult or impossible.

1.3 Plan B

The “Plan B” method is essentially to try and guess the answer - although the guessing is “intelligent”! This method is sometimes called “trial and error” although its official title is “trial and improvement” (because we don't make errors in maths).

1.4 Example (Trial and Improvement)

(a) Work out $x^3 + 2x - 33$ when x has the following values.

(i) $x = 10$

[2 marks]

(ii) $x = 0$

[2 marks]

(iii) $x = 1$

[2 marks]

(iv) $x = 3$

[2 marks]

(b) Which value of x is a solution to $x^3 + 2x - 33 = 0$?

[2 marks]

1.5 Example (Spreadsheet Search)

The spreadsheet shows $100x^3 + 200x^2 - 16x - 32$ for various values of x

x	$100x^3 + 200x^2 - 16x - 32$
0.0	-32.0
0.1	-31.5
0.2	-26.4
0.3	-16.1
0.4	00.0
0.5	22.5
0.6	52.0
0.7	89.1
0.8	134.4
0.9	188.5
1.0	252.0

State which of the values of x in the spreadsheet is the best solution to,

(i) $100x^3 + 200x^2 - 16x - 32 = 0$

[2 marks]

(ii) $100x^3 + 200x^2 - 16x - 32 = 50$

[2 marks]

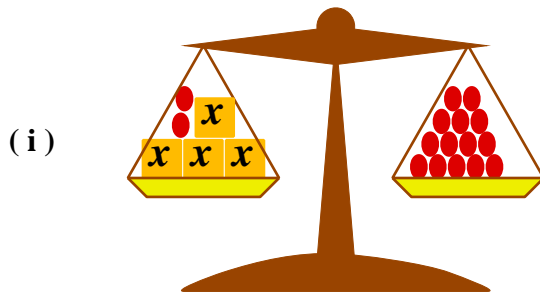
1.6 Exercise

You may use a calculator

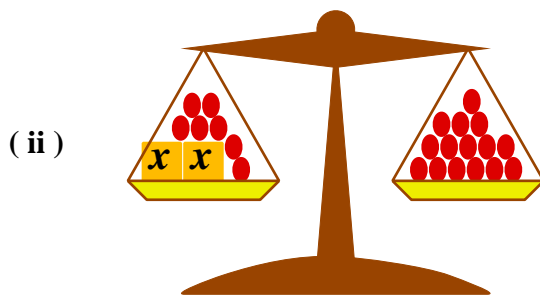
Marks Available : 50

Question 1

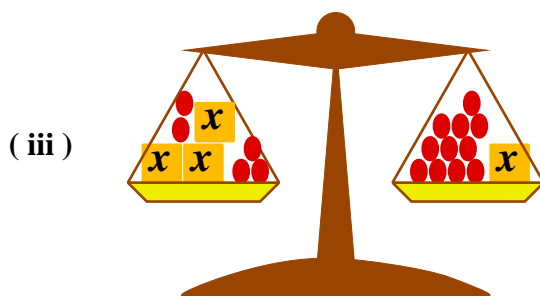
For each of the following balances, write down the associated equation and then solve it using the algebra of doing the same to both sides.



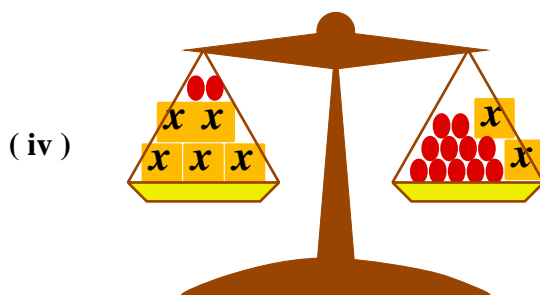
[3 marks]



[3 marks]



[3 marks]



[3 marks]

Question 2

(a) Work out $x^3 - 13x - 12$ when x has the following values.

(i) $x = 0$

[2 marks]

(ii) $x = 1$

[2 marks]

(iii) $x = 5$

[2 marks]

(iv) $x = 4$

[2 marks]

(b) Which value of x is a solution to $x^3 - 13x - 12 = 0$?

[2 marks]

Question 3

The spreadsheet shows $100x^3 + 100x^2 - x - 1$ for various values of x

x	$100x^3 + 100x^2 - x - 1$
0.0	-1.0
0.1	0.0
0.2	3.6
0.3	10.4
0.4	21.0
0.5	36.0
0.6	56.0
0.7	81.6
0.8	113.4
0.9	152.0
1.0	198.0

State which of the values of x in the spreadsheet is the best solution to,

(i) $100x^3 + 200x^2 - 16x - 32 = 0$

[2 marks]

(ii) $100x^3 + 200x^2 - 16x - 32 = 80$

[2 marks]

Question 4

(a) Work out $\frac{x^2 - \sqrt{x}}{2}$ when x has the following values.

(i) $x = 9$

[2 marks]

(ii) $x = 0$

[2 marks]

(iii) $x = 4$

[2 marks]

(iv) $x = 1$

[2 marks]

(b) Which values of x are solutions to $\frac{x^2 - \sqrt{x}}{2} = 0$

[2 marks]

Question 5

The spreadsheet shows $\cos(x) + \sin(x)$ for various values of x (in degrees).

x°	$\cos(x) + \sin(x)$
40	1.409
60	
80	1.158
100	0.811
120	0.366
140	-0.123
160	-0.598

The value corresponding to $x = 60^\circ$ is missing.

(i) Work out the missing value giving your answer to three decimal places.

[2 mark]

(ii) State which of the value of x in the spreadsheet is the best solution to,
 $\cos(x) + \sin(x) = 0$

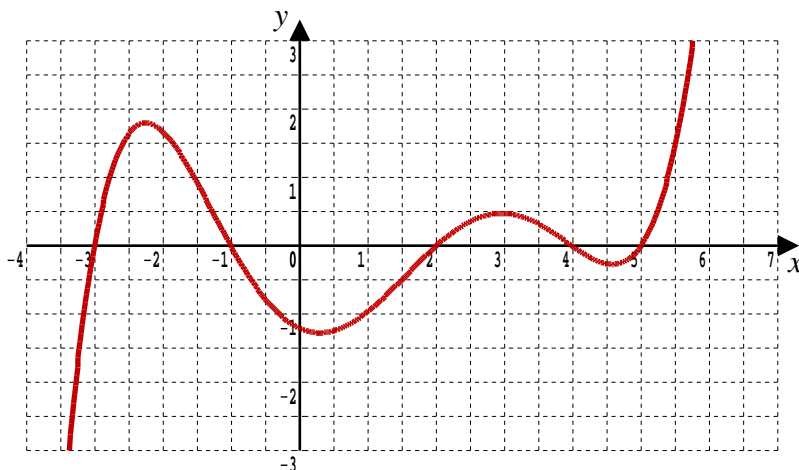
[2 mark]

Question 6

In this question an equation is solved by plotting the graph of the equation in a graph plotter on a computer.

The red curve is a plot of the the equation,

$$y = 0.01x^5 - 0.07x^4 - 0.03x^3 + 0.79x^2 - 0.46x - 1.2$$



- (i) From looking at the graph write down the five different integer values of x for which the equation equals zero.

In other words, write down the solutions to,

$$0.01x^5 - 0.07x^4 - 0.03x^3 + 0.79x^2 - 0.46x - 1.2 = 0$$

[5 marks]

- (ii) Draw the line with equation $y = 1$ on the graph.
From looking at the graph write down, correct to 1 decimal place, the three different values of x for which,

$$0.01x^5 - 0.07x^4 - 0.03x^3 + 0.79x^2 - 0.46x - 1.2 = 1$$

[5 marks]