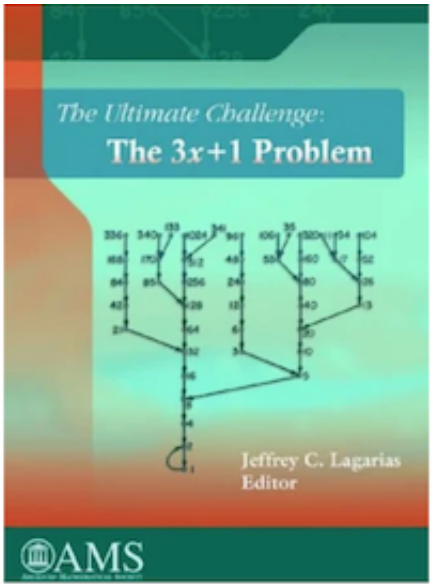
4.1 The Sign Flipped Collatz Rule

Entire books have been written about the Collatz conjecture. The current best was published in 2010 and is titled *The Ultimate Challenge: The 3x+1 Problem.* It is written by Jeffrey Lagarias who is a mathematics professor at the University of Michigan, USA, and features a few guest chapters by other mathematicians who have spent time working on this problem.



The Ultimate Challenge: The 3x+1 Problem [Book] £54.50
Hardback · Non-fiction Amazon.co.uk

In 2021 an updated version of the first chapter was made available free of charge on the arXiv website: https://arXiv.org/abs/2111.02635

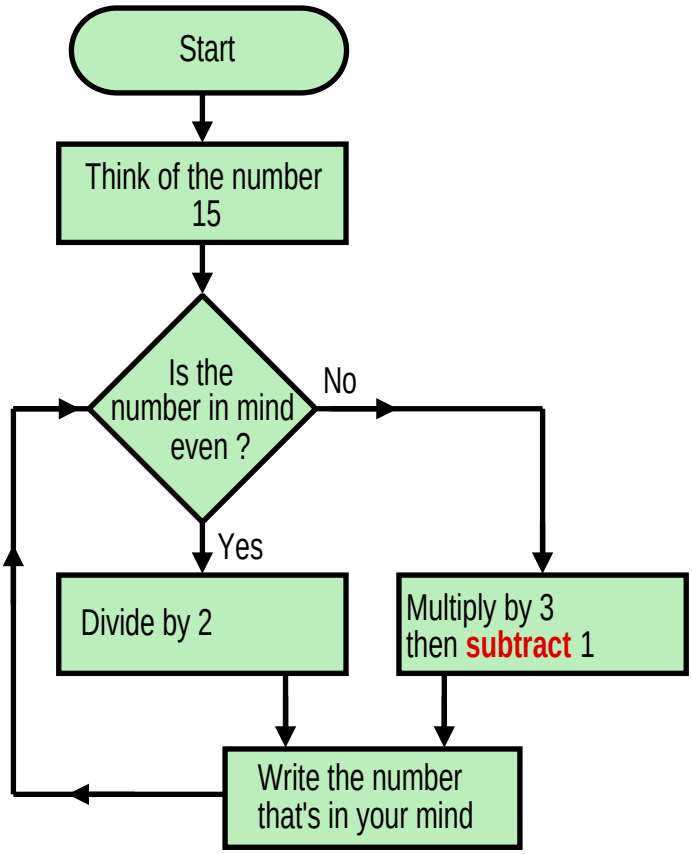
There is a good Numberphile video that summarises the Collatz Conjecture featuring Professor David Eisenbud (about 8 minutes).

Teaching Video : https://www.NumberWonder.co.uk/v9120/2.mp4

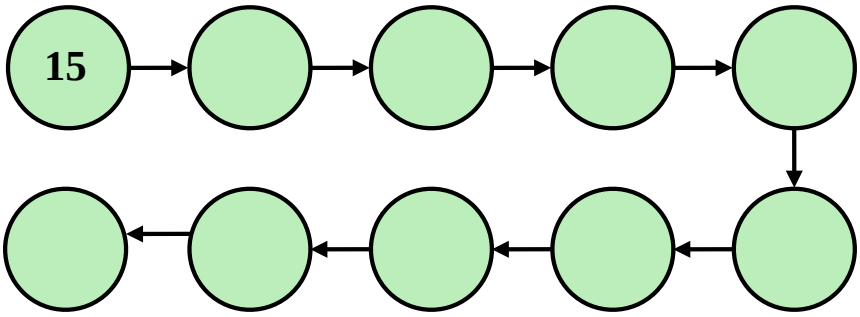
Towards the end of the video there is a suggestion that a similar iteration be looked at, *the sign flipped Collatz rule*, which is what our last lesson on the topic is about.

4.2 If At First You Don't Succeed

When a problem seems difficult, one avenue to explore is that of similar problems to see what sort of things happen there. Do they exhibit similar behaviours or do different things occur ?



On the diagram write the number sequence generated by the flowchart.



[4 marks]

This initial exploration of the *sign flipped Collatz rule*, given by,

$$y = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ 3x - 1 & \text{if } x \text{ is odd} \end{cases}$$

seems to suggest that it's likely to behave in much the same way as the original rule.

However, prepare for some surprises as you work through the exercise !

4.3 Exercise

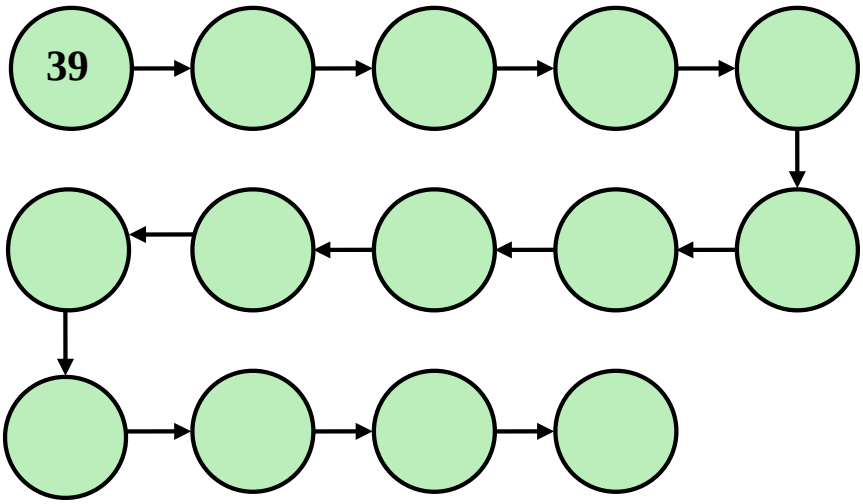
Marks Available : 38

Question 1

Use the *sign flipped Collatz rule*, given by,

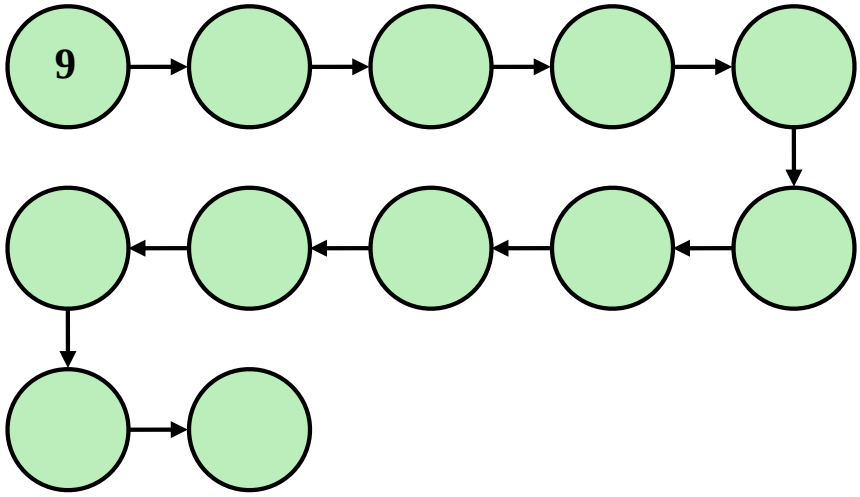
$$y = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ 3x - 1 & \text{if } x \text{ is odd} \end{cases}$$

(i) with a starting number of 39.



[4 marks]

(ii) with a starting number of 9.



[4 marks]

(iii) Comment on your answers.

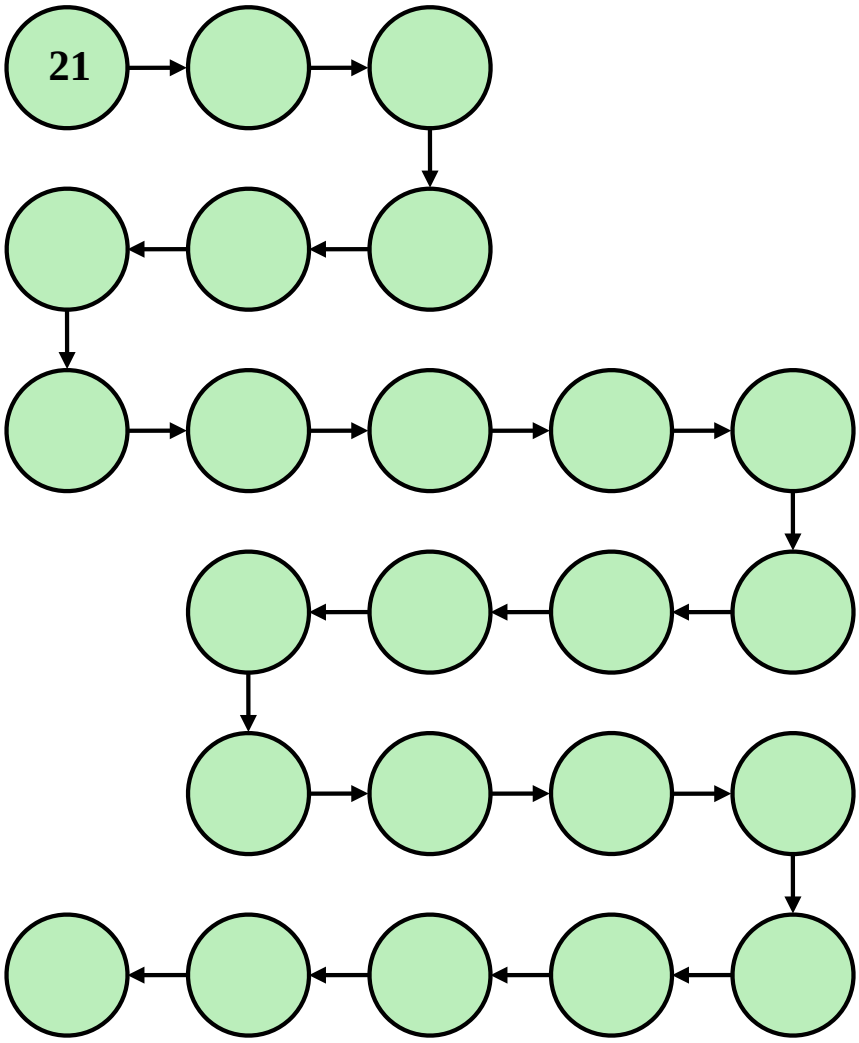
[2 marks]

Question 2

In this question we'll look a little more at the *sign flipped Collatz rule*,

$$y = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ 3x - 1 & \text{if } x \text{ is odd} \end{cases}$$

(i) Complete the following diagram.



[6 marks]

(ii) Explain how the diagram shows the *sign flipped Collatz rule* can have different behaviour to the regular rule used by the Collatz conjecture.

[2 marks]

Question 3

Watts' Conjecture

Here is an iterative rule that's a variation on those previously looked at.

$$y = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ 5x - 3 & \text{if } x \text{ is odd} \end{cases}$$

Watts' Conjecture claims that all positive integers will either

(a) Terminate in the 1-loop [1, 2]

or

(b) terminate in the 3-loop [3, 12, 6]

To get a good initial feel for this iterative rule, work through the sequence generated by starting from,

(i) The number 5

[4 marks]

(ii) The number 15

[4 marks]

(iii) The numbers less than 100 so far have partitioned as follows;

Watts' 100 Table

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

An interesting pattern seems to emerge if the start numbers that go to the 1-loop are coloured differently to those going to the 3-loop. Investigate if this pattern continues to hold.

[12 marks]