3.1 The Monster 27 Sequence

Our *Collatz 100 Table* after two lessons looks like this;

Collatz 100 Table

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

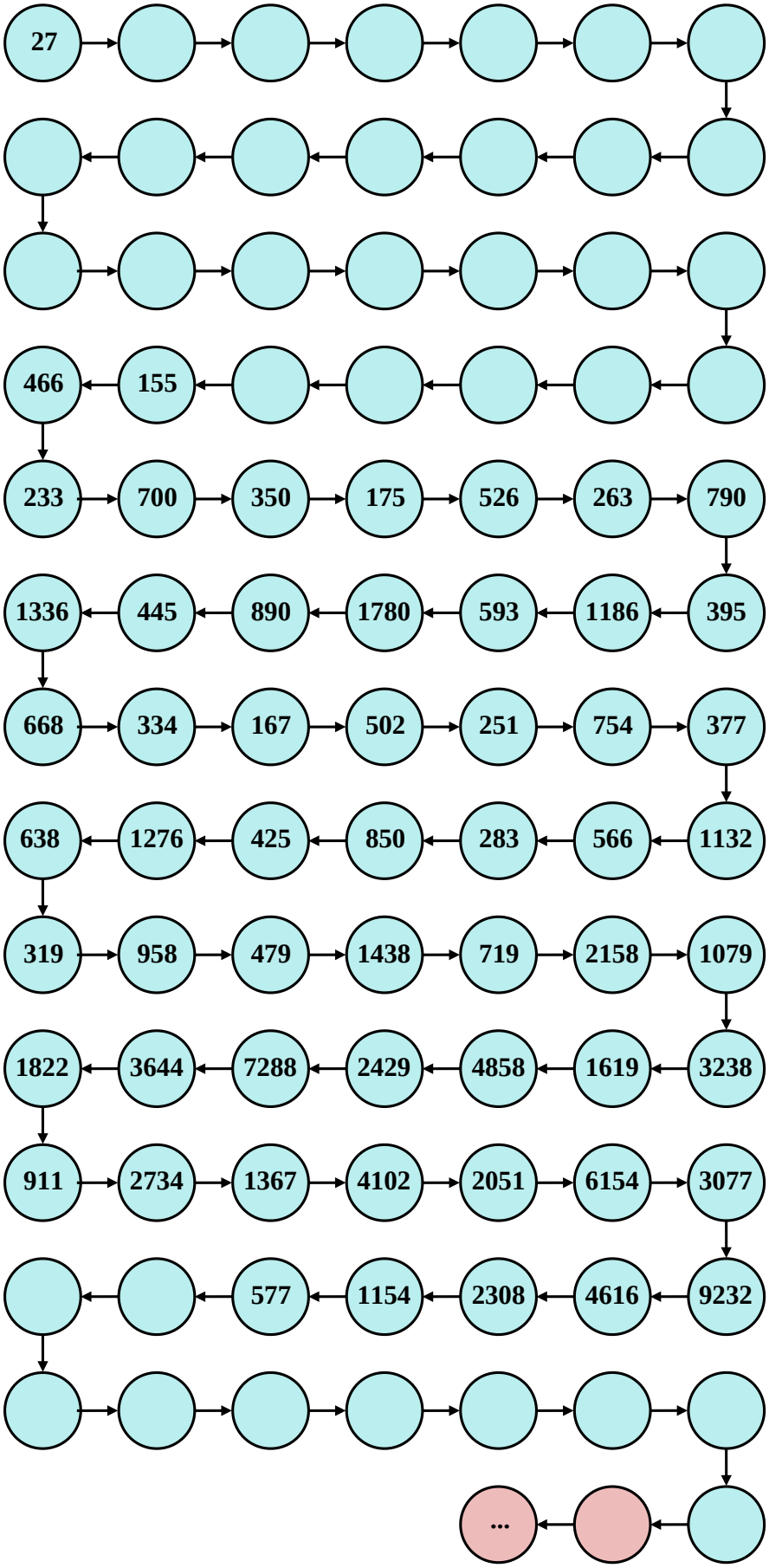
The obvious next number to try is 27.

If you have written your own computer program to generate Collatz sequences use it to obtain the 27 sequence. Otherwise use the online Collatz calculator at;

https://goodcalculators.com/collatz-conjecture-calculator/

Complete the 27 sequence over the page.

- You stop once you get to a number for which you know how the story ends.
- Cross out the appropriate numbers in *Collatz 100 Table* (above).
- Don't forget to also cross out the 27.


27
466
155
233
700
350
175
526
263
790
1336
445
890
1780
593
1186
395
668
334
167
502
251
754
377
638
1276
425
850
283
566
1132
319
958
479
1438
719
2158
1079
1822
3644
7288
2429
4858
1619
3238
911
2734
1367
4102
2051
6154
3077
577
1154
2308
4616
9232
...

[8 marks]

3.2 Exercise

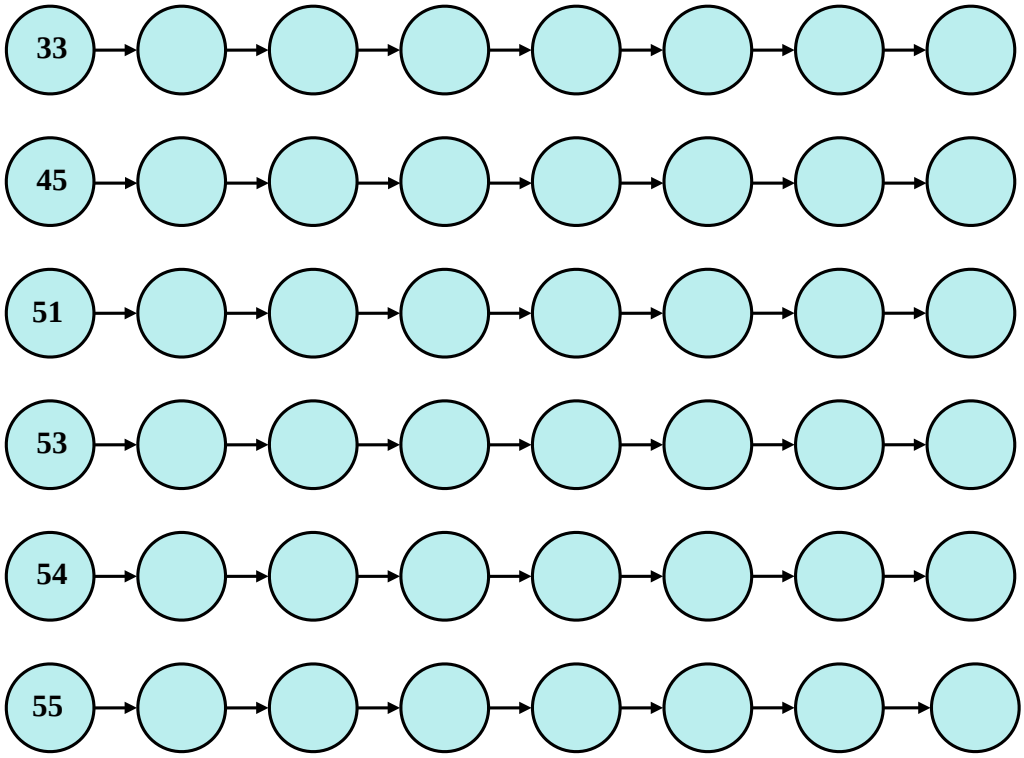
Marks Available : 44

Question 1

Completing the *Collatz 100 Table*

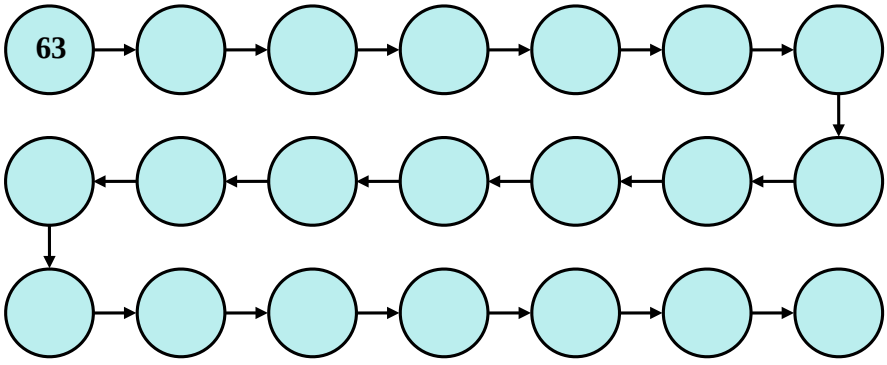
Use a Collatz calculator to systematically check the numbers that have not yet been accounted for in the Collatz 100 Table. Do this with the aid of the following diagrams. You will not need to fill in all of the circles each time because as soon as you get a number that is already accounted for in the table you know how the story ends.

(i) Starting to working through the numbers not yet accounted for...



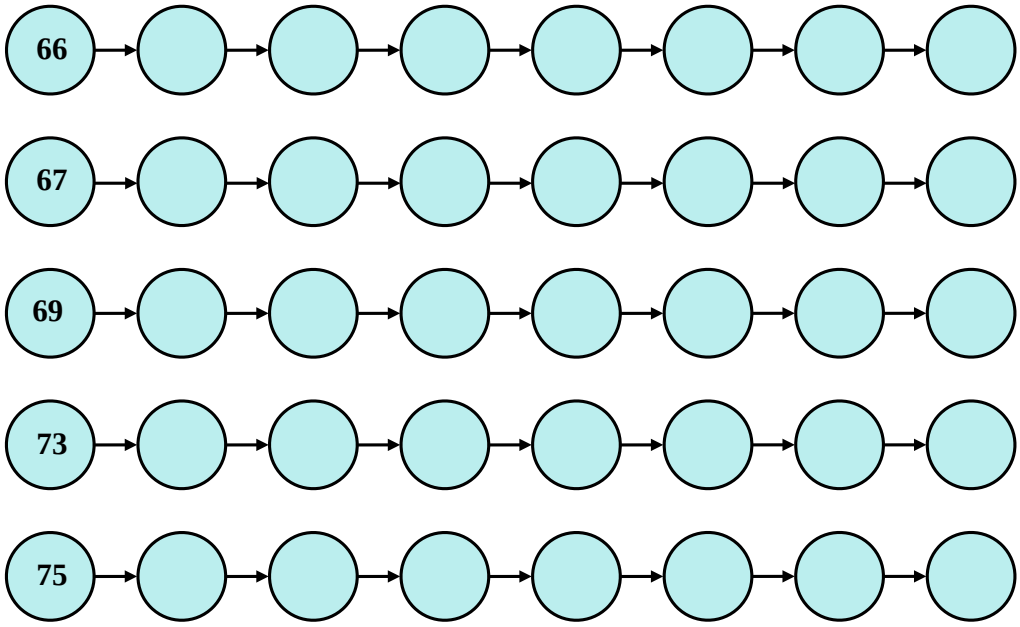
[6 marks]

(ii) A longer sequence...



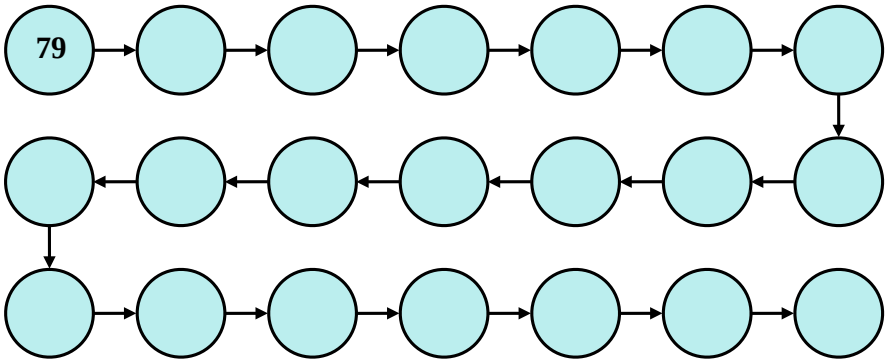
[3 marks]

(iii) Some more that join on to known numbers quickly...



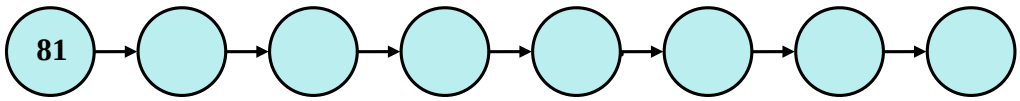
[5 marks]

(iv) Another longer sequence ...



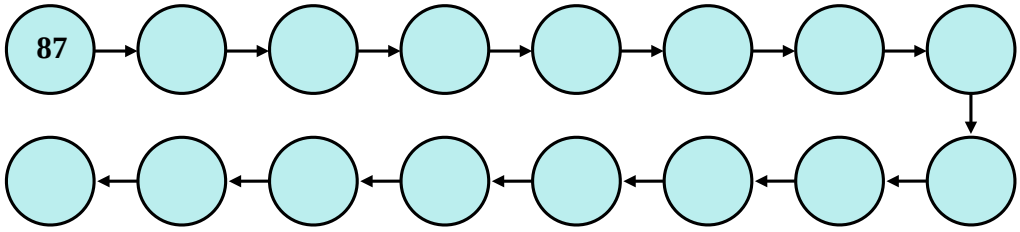
[3 marks]

(v) Short ...



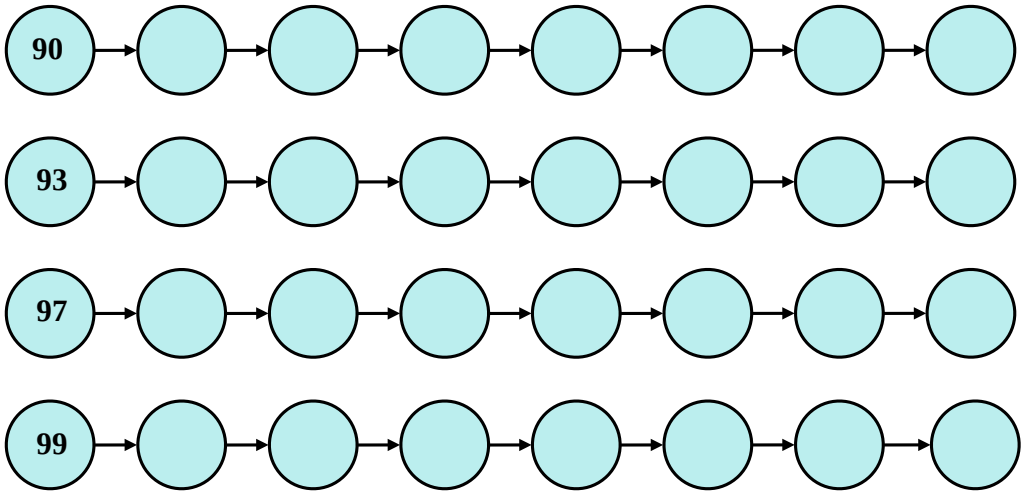
[1 mark]

(vi) Long ...



[2 marks]

(vii) Going for the finish ...



[4 marks]

Question 2

The *Total Stopping Time* of a Collatz sequence is the number of times the starting number is iterated before reaching the number 1.

For example, the number 8 has a *Total Stopping Time* of 3 because from the number 8, the first iteration gives 4, the second iteration gives 2 and the third iteration gives 1.

The online Collatz calculator gives the *Total Stopping Time*.

Here are the *Total Stopping Times* for the *Collatz 100 Table*;

Total Stopping Times 1 - 100

	1	2	3	4	5	6	7	8	9	10
00	0	1	7	2	5	8	16	3	19	6
10	14	9	9	17	17	4	12	20	20	7
20	7	15	15	10	23	10	111	18	18	18
30	106	5	26	13	13	21	21	21	34	8
40	109	8	29	16	16	16	104	11	24	24
50	24	11	11	112	112	19	32	19	32	19
60	19	107	107	6	27	27	27	14	14	14
70	102	22	115	22	14	22	22	35	35	9
80	22	110	110	9	9	30	30	17	30	17
90	92	17	17	105	105	12	118	25	25	25

In the *Total Stopping Times 1- 100* table,

(i) What is the longest stopping time ?

[2 marks]

(ii) Which starting number has this longest stopping time ?

[2 marks]

Amongst mathematicians there is considerable interest in these *Total Stopping Times*. For example, you may have noticed that you often get two and sometimes three different starting numbers next to each other with the same *Total Stopping Time*. This raises the question, can you get four, five, six, ... consecutive starting values with the same *Total Stopping Time* ?

(iii) Complete the following Total Stopping Time table for starting numbers between 101 and 200. You could write a computer program to do this or use the online Collatz calculator.

Total Stopping Times 101 - 200

	1	2	3	4	5	6	7	8	9	10
100										
110										
120										
130										
140										
150										
160										
170										
180										
190										

[12 marks]

(iv) Can you get four, five, six, ... consecutive starting values with the same *Total Stopping Time* ?

[4 marks]