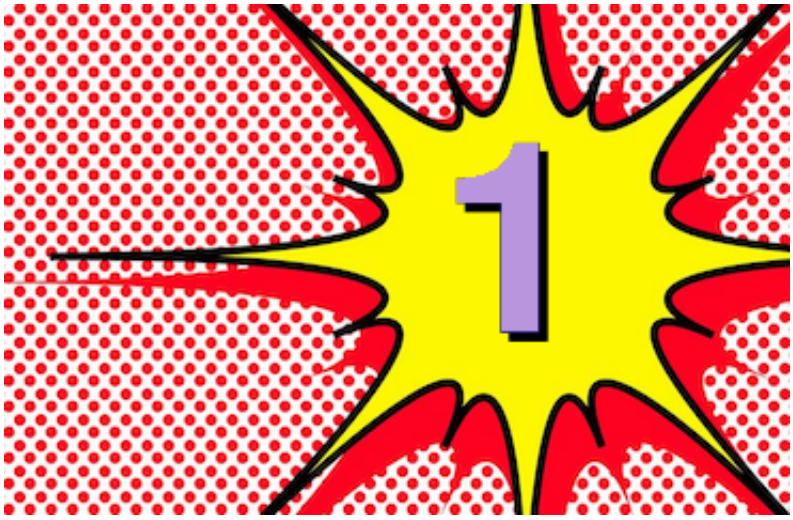
6.1 Around and Around



Last lesson we started to have a deeper appreciation of just how useful the number one can be in allowing us to handle some tricky calculations involving fractions without resorting to the use of a calculator. In this lesson we look at a variation of the same theme. Once again, the objective is to go around and around in an iterative loop.

6.2 Example

Without using a calculator, simplify the following,

$$\frac{1}{\left(\frac{3}{4}+1\right)}$$

There are many ways of tackling this but we are going to seemingly randomly, multiply it by one. This is allowed because it does not change anything. But although it's allowed why do it ?

$$\frac{1}{\left(\frac{3}{4}+1\right)} \times 1$$

The 4 is coloured red because we're going to use the red number method of doing the calculation. This involves picking the red face of the number one.

[2 marks]

6.3 Example

This work is still non-calculator, so we will make heavy use of the red number method and the many faces of the number one to not get into a muddle with all of the fractions involved.

Consider the iteration, $F_1 = 3$, $F_{n+1} = \dfrac{2}{F_n + 1}$

Use the space below to work out the first five terms of this iterative sequence and put your answers in the table at the bottom of the page.

F_1	F_2	F_3	F_4	F_5	F_6

[7 marks]

6.4 Exercise

Non-Calculator

Marks Available : 40

Question 1

Each of the following calculations has an answer that is a rational number.

That is, a number in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$

For each, determine what that rational number is.

(i) $\dfrac{1}{\left(\frac{2}{3} + 5\right)} \times \dfrac{3}{3}$

[2 marks]

(ii) $\dfrac{4}{\left(\frac{5}{2} + 3\right)} \times \dfrac{2}{2}$

[2 marks]

(iii) $\dfrac{3}{\left(\frac{9}{4} - 1\right)} \times \dfrac{4}{4}$

[2 marks]

(iv) $\dfrac{4}{\left(\frac{8}{13} + 3\right)} \times \dfrac{13}{13}$

[2 marks]

Question 2

Each of the following calculations has an answer that is a rational number.

That is, a number in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$

For each, determine what that rational number is.

Use the "red number" technique suggested by question 1.

(i) $\dfrac{1}{\left(\frac{4}{3} + 2\right)}$

[2 marks]

(ii) $\dfrac{5}{\left(\frac{3}{7} + 2\right)}$

[2 marks]

(iii) $\dfrac{4}{\left(\frac{23}{20} + 5\right)}$

[2 marks]

(iv) $\dfrac{7}{\left(\frac{100}{101} + 1\right)}$

[2 marks]

Question 3

(i) The following sum has an answer that is a rational number.

That is, a number in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$

Determine what that rational number is.

$$\frac{1}{\left(\frac{1}{2} + 1\right)}$$

[2 marks]

(ii) Consider the iteration, $A_1 = \frac{1}{2}$, $A_{n+1} = \frac{1}{A_n + 1}$

Use the space below to work out the first six terms of this iterative sequence and put your answers in the table towards the bottom of the page.

A_1	A_2	A_3	A_4	A_5	A_6

[7 marks]

(iii) From looking at your table of results, and spotting a pattern, write down what the next three terms, A_7, A_8 and A_9 are likely to be.

[2 marks]

Question 4

(i) The following sum has an answer that is a rational number.

That is, a number in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$

Determine what that rational number is.

$$\frac{1}{\left(\frac{2}{3} + 1\right)}$$

[2 marks]

(ii) Consider the iteration, $B_1 = \frac{1}{3}$, $B_{n+1} = \frac{1}{2B_n + 1}$

Use the space below to work out the first six terms of this iterative sequence and put your answers in the table towards the bottom of the page.

B_1	B_2	B_3	B_4	B_5	B_6

[7 marks]

(iii) From looking at your table of results, and spotting a pattern, write down what the next three terms, B_7, B_8 and B_9 are likely to be.

[2 marks]

Question 5

(i) The following sum has an answer that is a rational number.

That is, a number in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$

Determine what that rational number is.

$$\frac{1}{\left(\frac{3}{5} + 2\right)}$$

[2 marks]

(ii) Consider the iteration, $Z_1 = \frac{1}{5}$, $Z_{n+1} = \frac{1}{3Z_n + 2}$

Use the space below to work out the first six terms of this iterative sequence and put your answers in the table towards the bottom of the page.

Z_1	Z_2	Z_3	Z_4	Z_5	Z_6

[8 marks]

(iii) From looking at your table of results, and spotting a pattern, write down what the next three terms, Z_7, Z_8 and Z_9 are likely to be.

[2 marks]