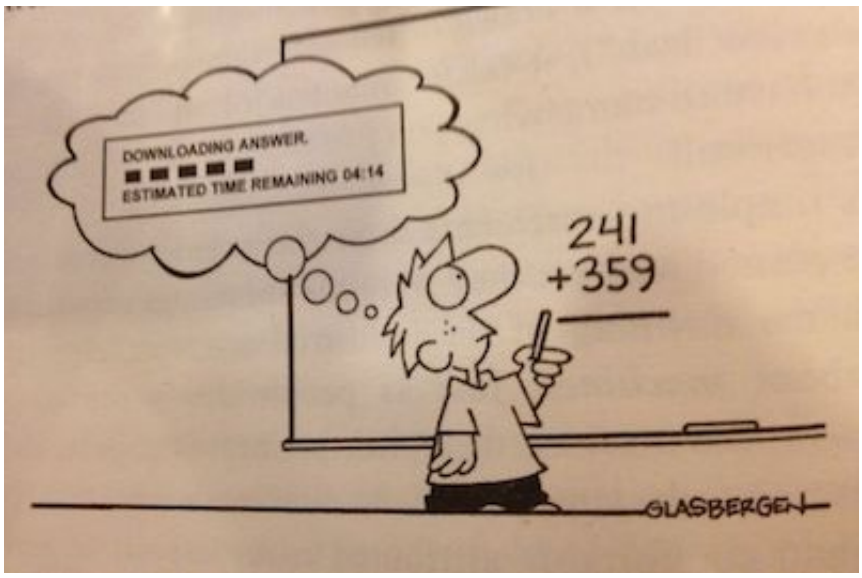
Lesson 4

GCSE Mathematics
Iteration

4.1 Fraction Frolics



To date, our iterations been restricted to numbers that are integers, $\mathbb{Z}$.

$$\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

However, in this lesson we will look at iterations that live amongst the rational numbers, $\mathbb{Q}$. Examples of rational numbers include,

$$\frac{2}{3}, \frac{7}{2}, \frac{6}{1}, -\frac{1}{3}, \frac{0}{1}$$

A rational number is a number that can be written as one integer divided by another (but not divided by zero, as division by zero is not allowed).

4.2 Examples #1
(Non-Calculator)

Calculate and simplify,

(i) $\frac{3}{7} \times \frac{4}{5}$ **(ii)** $\frac{5}{12} \times \frac{4}{7}$

(iii) $\frac{4}{7} \times 2$ **(iv)** $\frac{12}{13} \times 13$

[4 marks]

4.3 Examples #2
(Non-Calculator)

Calculate and simplify,

(i) $12\left(\frac{1}{4} + 2\right)$ **(ii)** $\left(\frac{4}{5} + \frac{1}{3}\right) \times 15$

(iii) $8\left(2 + \frac{3}{8}\right)$ **(iv)** $\left(\frac{3}{5} + 2\right) \times 5$

[4 marks]

4.4 An Iteration Involving Fractions

A sequence of numbers has the iterative rule

$$U_1 = \frac{3}{5} \qquad U_{n+1} = \frac{1}{2} U_n$$

Use the space below to work out the first six terms of this iterative sequence and put your answers in the table at the bottom of the page.

U_1	U_2	U_3	U_4	U_5	U_6

[4 marks]

4.5 Exercise

Non-Calculator

Marks Available : 50

Question 1

Calculate and simplify,

(i) $\frac{3}{11} \times \frac{5}{7}$ **(ii)** $\frac{4}{9} \times \frac{3}{5}$

(iii) $\frac{5}{13} \times 2$ **(iv)** $\frac{10}{17} \times 17$

[4 marks]

Question 2

First expand the brackets, then simplify,

(i) $15\left(\frac{1}{5} + 2\right)$ **(ii)** $\left(\frac{3}{4} + \frac{1}{6}\right) \times 12$

(iii) $7\left(3 + \frac{4}{7}\right)$ **(iv)** $\left(\frac{5}{6} + 2\right) \times 6$

[4 marks]

Question 3

First expand the brackets, then simplify,

(i) $\frac{2}{3}\left(2 + \frac{1}{2}\right)$ **(ii)** $\frac{3}{5}\left(\frac{4}{3} + 3\right)$

[4 marks]

Question 4

A sequence of numbers has the iterative rule

$$A_1 = \frac{1}{2} \qquad A_{n+1} = \frac{1}{2} A_n$$

Use the space below to work out the first six terms of this iterative sequence then put your answers in the table.

A_1	A_2	A_3	A_4	A_5	A_6

[6 marks]

Question 5

A sequence of numbers has the iterative rule

$$B_1 = \frac{16}{81} \qquad B_{n+1} = \frac{3}{2} B_n$$

Use the space below to work out the first eight terms of this iterative sequence then put your answers in the table.

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8

[6 marks]

Question 6

First expand the brackets, then simplify,

(i) $3\left(2 + \frac{1}{3}\right)$

[2 marks]

(ii) $\frac{3}{3} \times \frac{\left(2 + \frac{1}{3}\right)}{1}$

[2 marks]

Question 7

First expand the brackets, then simplify,

(i) $5\left(3 + \frac{2}{5}\right)$

[2 marks]

(ii) $\frac{5}{5} \times \frac{\left(3 + \frac{2}{5}\right)}{1}$

[2 marks]

Question 8

First expand the brackets, then simplify,

(i) $\left(2 + \frac{3}{7}\right) \times 7$

[2 marks]

(ii) $\frac{\left(2 + \frac{3}{7}\right)}{1} \times \frac{7}{7}$

[2 marks]

Question 9

Calculate $4 + \frac{3}{5}$ by

- Putting brackets around the question
- Putting the bracketed question all over 1
- Multiplying by $\frac{5}{5}$

[4 marks]

Question 10

Calculate $3 + \frac{4}{7}$ by

- Putting brackets around the question
- Putting the bracketed question all over 1
- Multiplying by $\frac{7}{7}$

[4 marks]

Question 11

A sequence of numbers has the iterative rule

$$G_1 = \frac{1}{4} \qquad G_{n+1} = \frac{2}{3} G_n$$

Use the space below to work out the first six terms of this iterative sequence then put your answers in the table.

G_1	G_2	G_3	G_4	G_5	G_6

[6 marks]