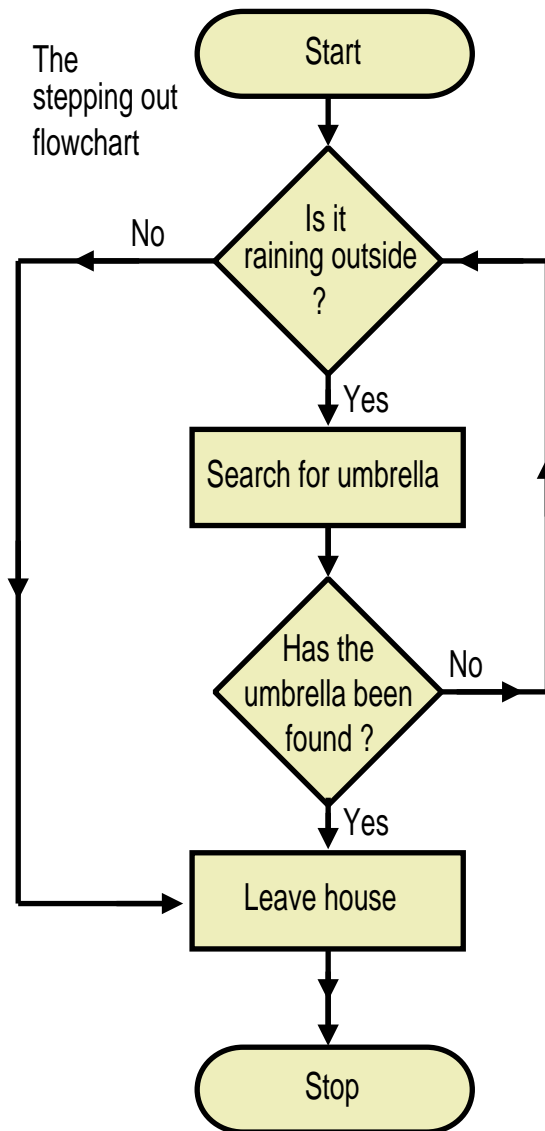


**2.1 Decisions, Decisions**

The following flowchart includes an extra feature in the form of a couple of decision diamonds. When a decision is to be made only two answers to the question are allowed; one of either “yes” or “no”.



- (i) What happens if it keeps raining and the umbrella cannot be found ?

[ 1 mark ]

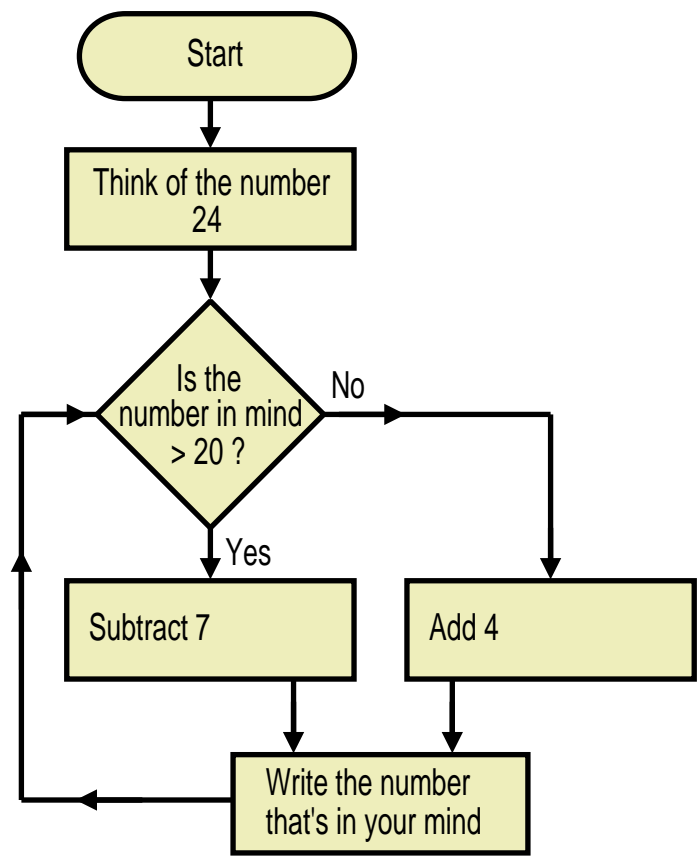
- (ii) Is it possible to leave the house when it is not raining but with the umbrella ? Explain your answer.

[ 2 marks ]

( iii ) How might the issues raised by parts (i) and (ii) be resolved ?

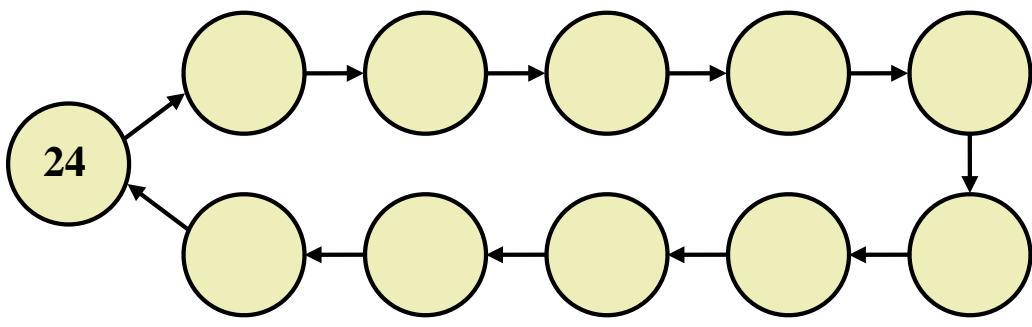
[ 2 marks ]

2.2 Example



The flowchart generates a loop of numbers.

On the following diagram write out the numbers that are in the loop.

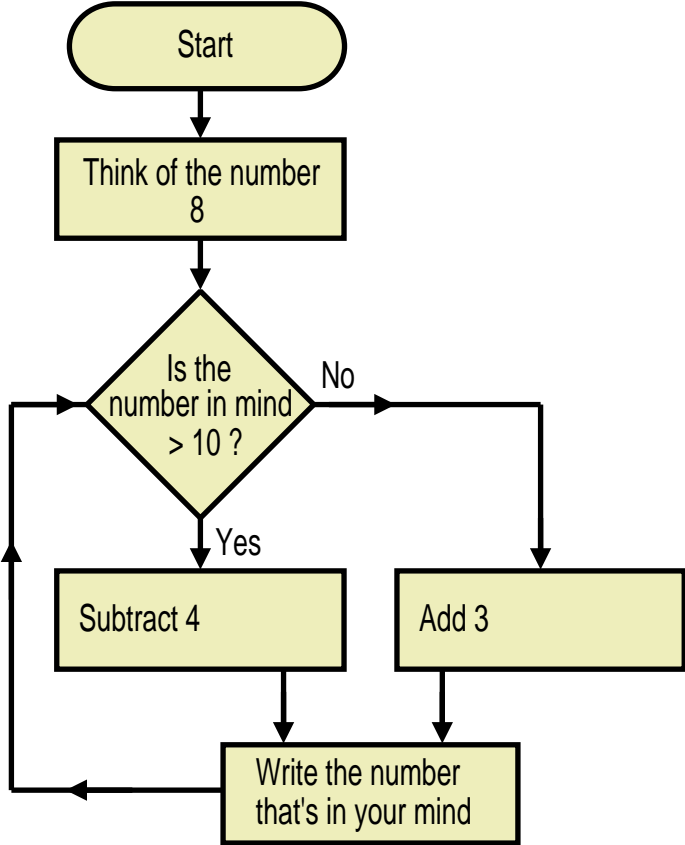


[ 6 marks ]

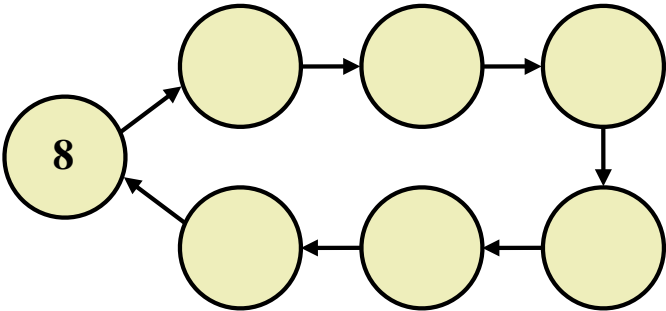
2.3 Exercise

**Non-Calculator**  
Marks Available : 40

Question 1

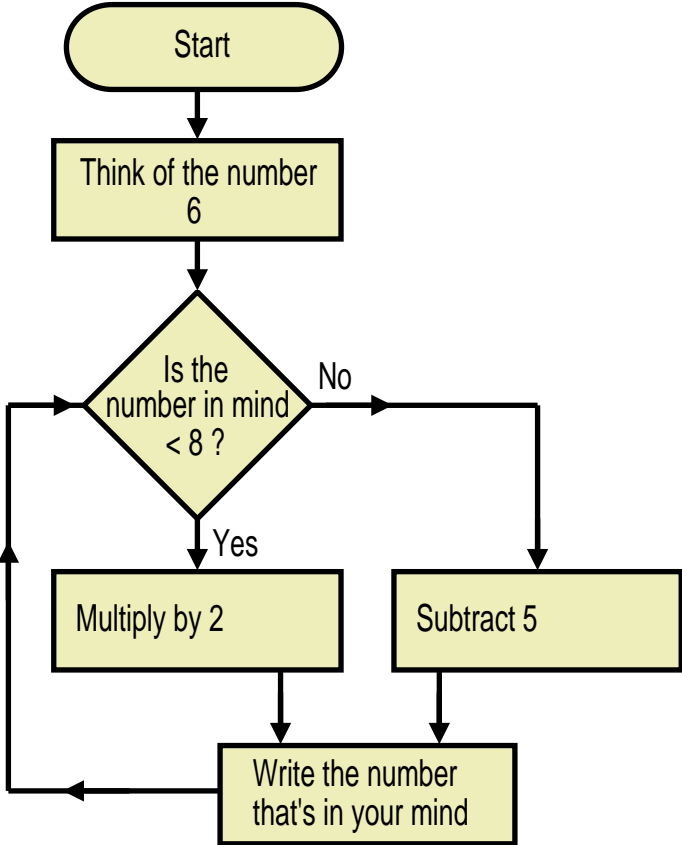


The following flowchart generates a loop of numbers.  
On the following diagram write out the numbers that are in the loop.

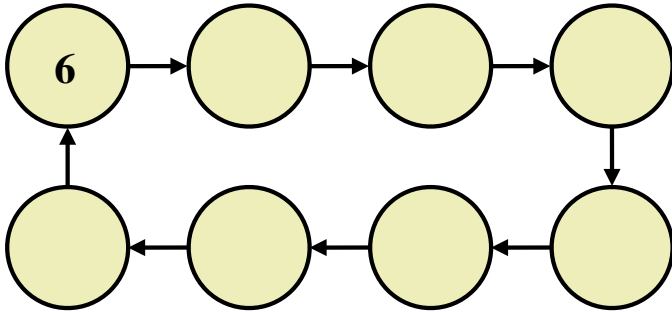


[ 6 marks ]

**Question 2**

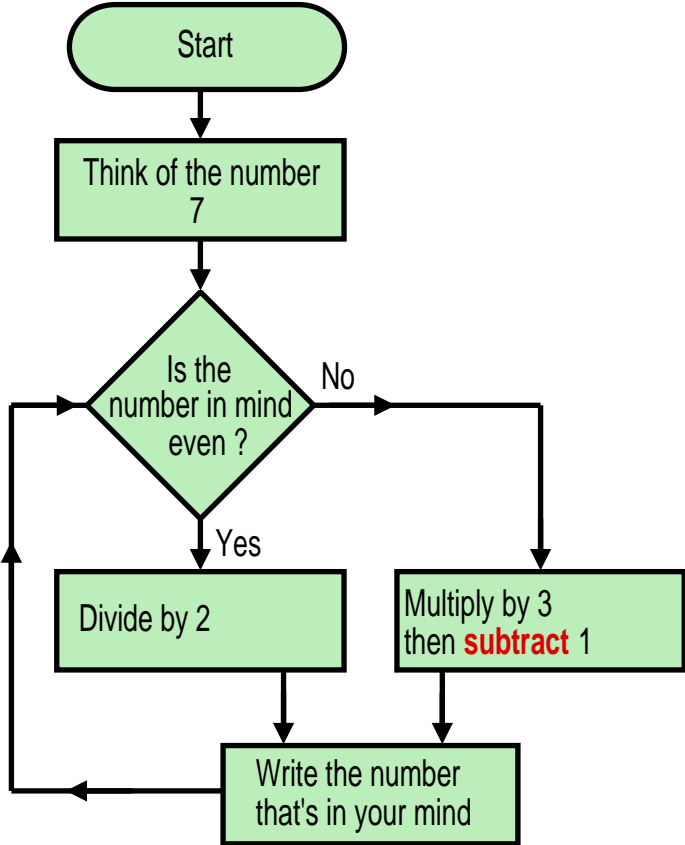


The following flowchart generates a loop of numbers.  
On the following diagram write out the numbers that are in the loop.

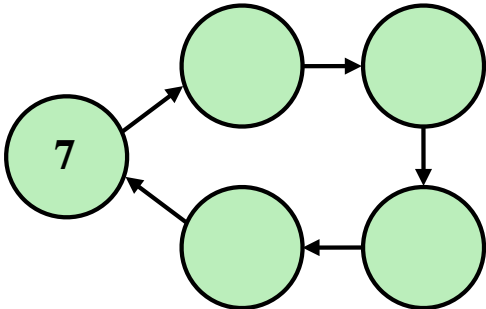


[ 6 marks ]

Question 3



The following flowchart generates a loop of numbers.  
On the following diagram write out the numbers that are in the loop.

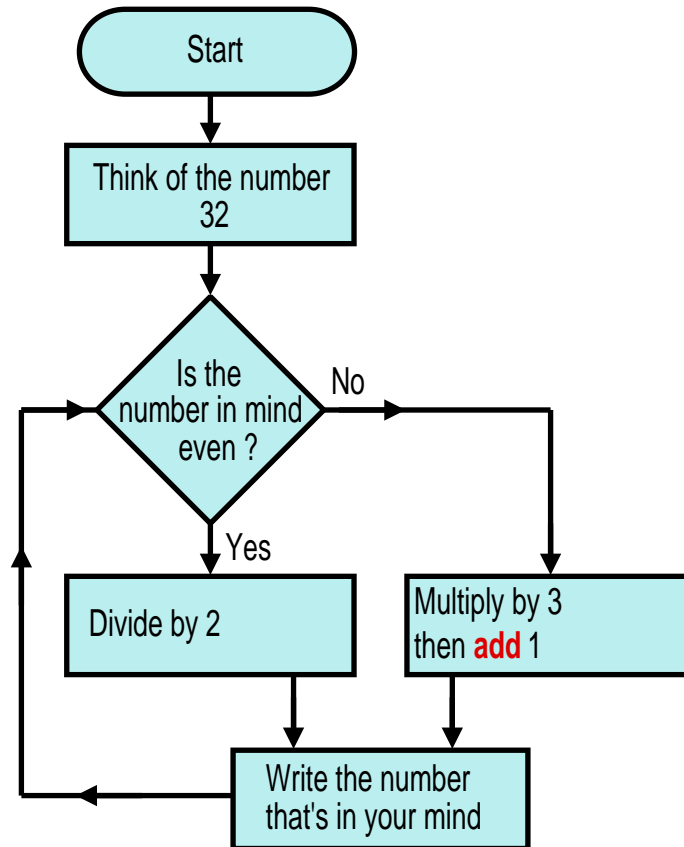


[ 6 marks ]

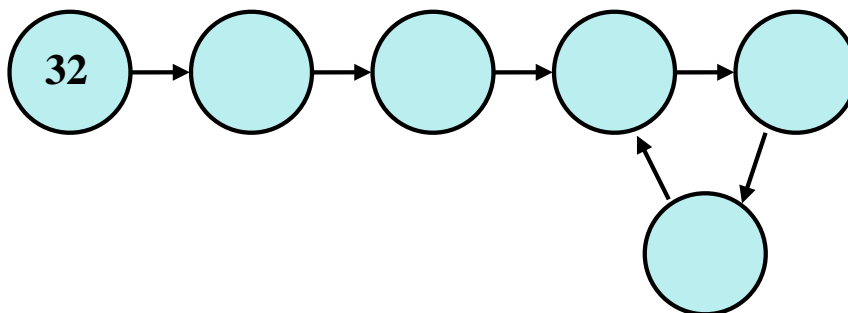
#### Question 4

(i)

Now to look at a problem that no-one has yet been able to solve since it was first stated in 1937 called the Collatz conjecture. It's about an iteration that the conjecture claims will always end up going around in a loop that includes the number 1. It will do this, so it is claimed, no matter what starting number is used. Here is the flowchart for the iteration with a starting number of 32.



On the following diagram write out the numbers that are generated by the flowchart.

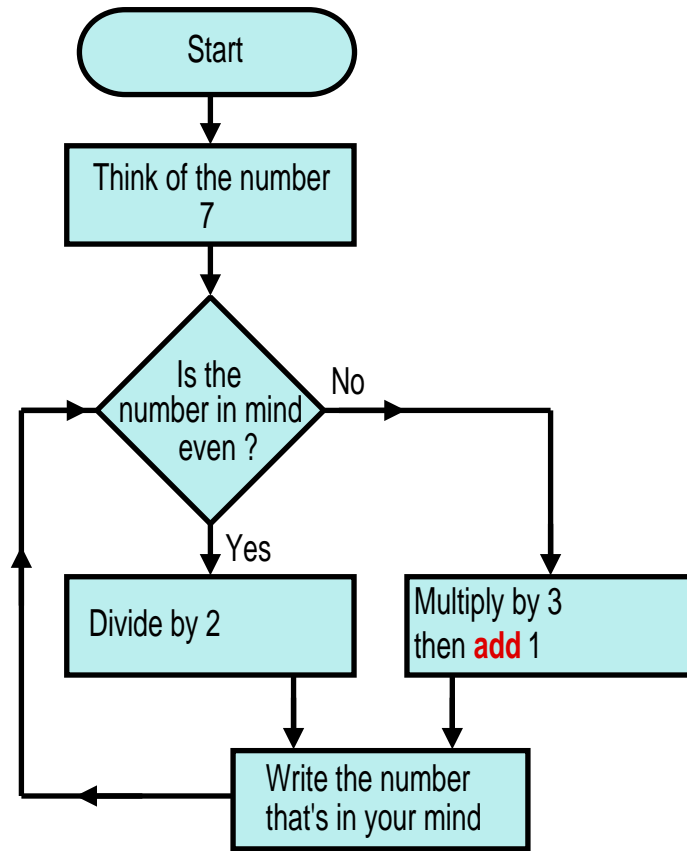


This is an example of a starting number that “falls to earth”.  
Not all numbers do this right away as you will discover next.

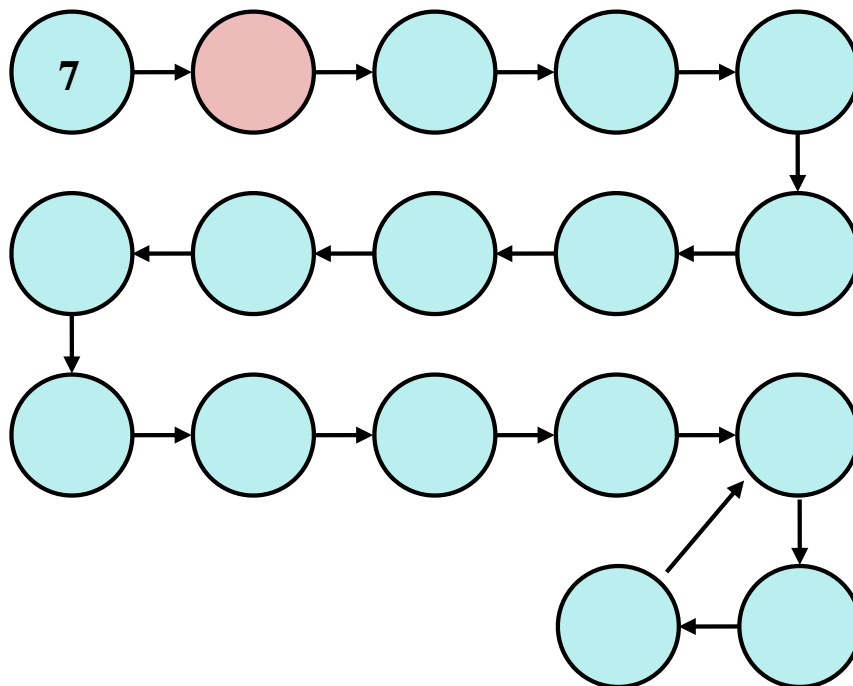
[ 4 marks ]

(ii)

Here is the flowchart for the same iteration but starting with the number of 7.



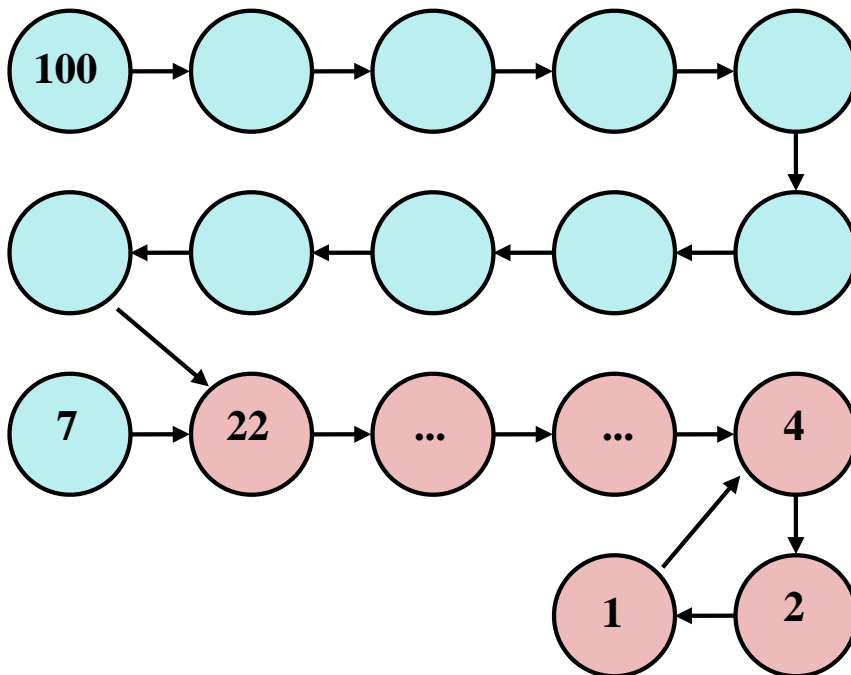
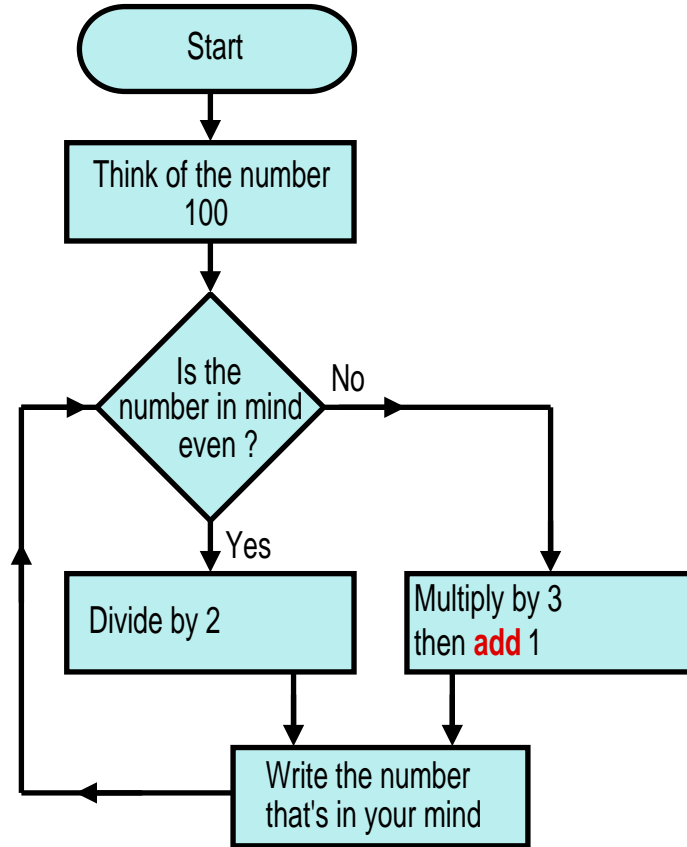
On the following diagram write out the numbers that are generated.



[ 8 marks ]

( iii )

From part (iii) you know what happens to many other starting numbers. For example, in the red circle you will have the number 22 which means that you know what happens if 22 is used as the start number. So, the only numbers worth trying now are numbers that did not occur in any of the circles of your part (i) or part (ii) answers. Let's try 100 next. Remember: No calculator !

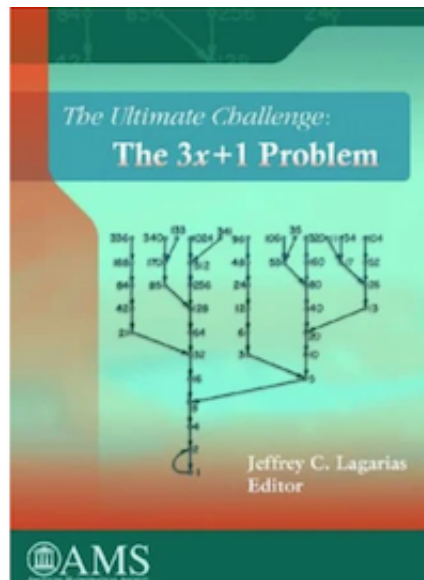


[ 8 marks ]



(iv)

Entire books have been written about the Collatz conjecture. To prove the conjecture is not true you would only have to find a starting number for which the loop at the end does not include the number one. No-one has yet found such a loop but people are still searching because nobody has managed to prove that the end loop will always include the number one. If you could prove that the Collatz conjecture is true you would be on the front pages of the world's newspapers. Similarly if you found a starting number that showed it is false.



The Ultimate Challenge: The  $3x+1$   
Problem [Book]      £54.50  
Hardback · Non-fiction [Amazon.co.uk](https://www.amazon.co.uk)

Watch the following Numberphile video in which Professor David Eisenbud talks about the Collatz Conjecture (about 8 minutes).

Teaching Video : <https://www.NumberWonder.co.uk/v9120/2.mp4>



[ 2 mark ]

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In October 2020, Shrewsbury School was voted “**Independent School of the Year 2020**”

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Teachers may obtain detailed worked solutions to the exercises by email from [mhh@shrewsbury.org.uk](mailto:mhh@shrewsbury.org.uk)