

Lesson 7

Further A-Level Pure Mathematics, Core 2 Differential Equations II

7.1 Dark Art Particular Integrals

Previously, the following table of suggested particular integrals was given. It has working well with the second order nonhomogeneous differential equations so far considered.

Form of $f(x)$	Form of PI
u	U
$ux + v$	$Ux + V$
$u x^r + \dots + vx + w$	$U x^r + \dots + Vx + W$
$u \cos kx$	$U \cos kx + V \sin kx$
$v \sin kx$	$U \cos kx + V \sin kx$
$u \cos kx + v \sin kx$	$U \cos kx + V \sin kx$
$u e^{kx}$	$U e^{kx}$
$u e^{-kx}$	$U e^{-kx}$

However, finding a particular integral can sometimes be something of a dark art and what a skilled mathematician chooses may not be obvious to beginners.



There is one additional piece of “wisdom from the elders” that we will embrace.

Clash of Function : An “Advice” Algorithm

When solving an equation of the form,

$$a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x), \text{ where } a, b \text{ and } c \text{ are constants,}$$

and having obtained the complementary function,

if a piece, $g(x)$, of a proposed particular integral, $p(x)$, has already occurred in the complementary function, modify the particular integral by multiplying it by x . That is, replace $p(x)$ with $x p(x)$.

Repeat this process, if necessary, until there is no piece in common in any part of the complementary function or particular integral.

Note: this advice algorithm was already at work within the complementary function when a repeated root occurred.

7.2 Example

Find the general solution to the differential equation,

$$\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} = 12$$

[6 marks]

What do you call the sudden urge to solve a differential equation ?



Calculust !

7.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 40

Question 1

Find the general solution to the differential equation,

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = e^{-3x}$$

[8 marks]

Question 2

y satisfies the differential equation,

$$\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} = 2x^2 - x + 1$$

- (i) Find the complementary function for this differential equation.

[3 marks]

- (ii) Explain, with reference to your part (i) answer, why the “obvious” particular integral of $y = Ux^2 + Vx + W$ is not suitable.

[2 marks]

- (iii) By using a suitable particular integral, find the general solution.

[8 marks]

Question 3

Find the general solution to the differential equation,

$$\frac{d^2y}{dx^2} + 12 \frac{dy}{dx} + 36y = 6e^{-6x}$$

[9 marks]

Question 4

Find the general solution to the differential equation,

$$\frac{d^2y}{dx^2} + 16y = \cos 4x$$

[10 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School

It may be freely duplicated and distributed, unaltered, for non-profit educational use

In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

© 2023 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk