Lesson 9

A-Level Pure Mathematics : Year 2

Integration III

9.1 Revision

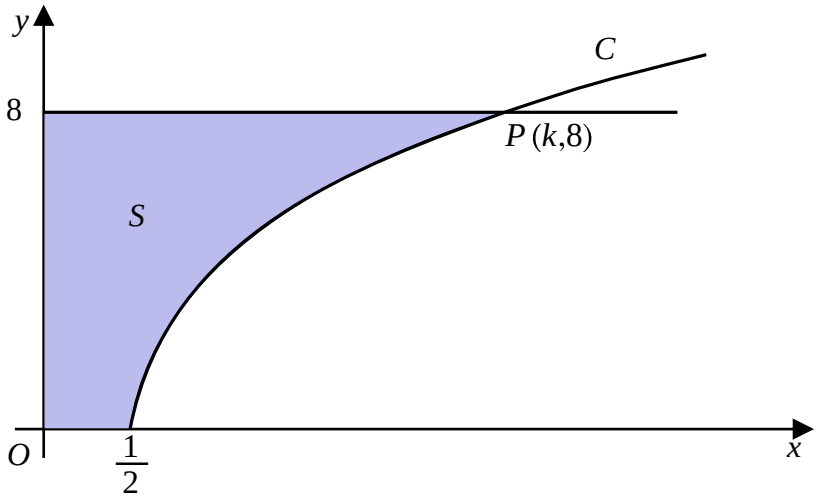
Show sufficient working to make your methods clear.

Marks Available : 40

Question 1

(a) Find $\int (2x - 1)^{\frac{3}{2}}\, dx$ giving your answer in its simplest form.

[3 marks]



The sketch shows part of the curve C with equation $y = (2x - 1)^{\frac{3}{2}}$, $x \geqslant \frac{1}{2}$ which cuts the line $y = 8$ at point P with coordinates $(k, 8)$, where k is a constant.

(b) Find the value of k

[2 marks]

(c) Find the shaded area, S, bounded by the coordinate axes, $y = 8$ and C.

[4 marks]

Question 2

A-Level Examination Question from October 2021, Paper 2, Q12 (Edexcel)

(a) Use the substitution $u = 1 + \sqrt{x}$ to show that,

$$\int_0^{16} \frac{x}{1 + \sqrt{x}}\, dx = \int_p^q \frac{2(u - 1)^3}{u}\, du$$

where p and q are constants to be found.

[3 marks]

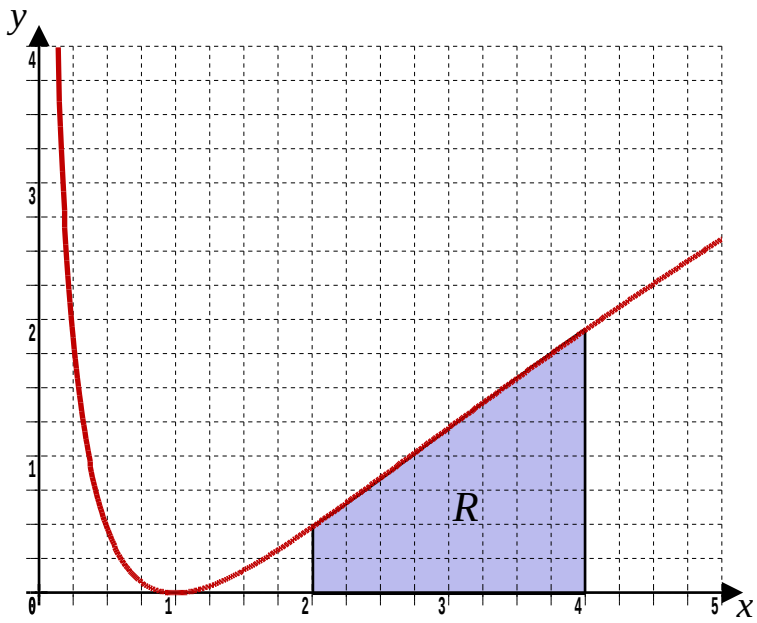
(b) Hence show that, $\int_0^{16} \frac{x}{1 + \sqrt{x}}\, dx = A - B\ln 5$

where A and B are constants to be found.

[4 marks]

Question 3

A-Level Examination Question from October 2021, Paper 1, Q11 (Edexcel)



The graph shows part of the curve with equation, $y = (\ln x)^2$, $x > 0$. The finite region R, shown shaded, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$. The table below shows corresponding values of x and y, with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R. giving your answer to 3 significant figures.

[3 marks]

(b) Use algebraic integration to find the exact area of R, giving your answer in the form,

$$y = a(\ln 2)^2 + b \ln 2 + c$$

where a, b and c are integers to be found.

[5 marks]

Question 4

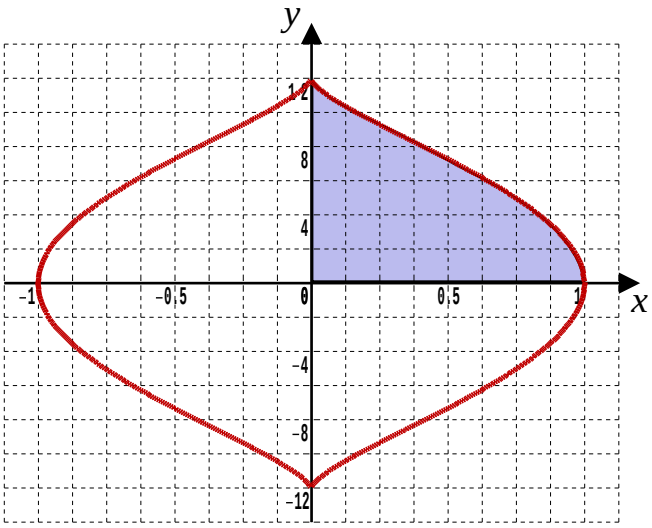
A-Level Examination Question from October 2021, Paper 1, Q14 (Edexcel)

Given that $y = \frac{x - 4}{2 + \sqrt{x}}$, $x > 0$, show that $\frac{dy}{dx} = \frac{1}{A\sqrt{x}}$, $x > 0$

where A is a constant to be found.

[4 marks]

Question 5



The graph is of the curve C with parametric equations,

$$x = \cos^3\theta, \quad y = 12\sin\theta, \quad 0 \leqslant \theta < 2\pi$$

The finite region in the first quadrant, bounded by C and the coordinate axes, is shown shaded. The curve is symmetrical in both the x and the y axis.

(a) Show that the area of the shaded region is given by the integral,

$$36\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\, d\theta$$

[4 marks]

(b) Use trigonometric identities to show that,

$$\cos^2\theta \sin^2\theta = \frac{1}{8}\left(1 - \cos 4\theta\right)$$

[4 marks]

(c) Hence find, in terms of π, the total area enclosed by C.

[4 marks]