Lesson 5

5.1 Year 2 Integration : Examination Questions

The formulae book provided in the examination gives the derivative of many functions. These are identified with a * in the table below, which also highlights several key results that are NOT provided. Used backward, the table gives integrals.

$f(x)$	$f'(x)$	Given ?
$sin\ x$	$cos\ x$	
$cos\ x$	$-\ sin\ x$	
$tan\ x$	$sec^2 x$	*
$sec\ x$	$sec\ x\ tan\ x$	*
$csc\ x$	$-\ csc\ x\ cot\ x$	*
$cot\ x$	$-\ csc^2 x$	*
$ln\ x$	$\frac{1}{x}$	
$ln\left|\ sec\ x\ \right|$	$tan\ x$	*
$ln\left|\ sin\ x\ \right|$	$cot\ x$	*
$ln\ \left|\ sec\ x\ +\ tan\ x\ \right|$	$sec\ x$	*
$ln\ \left|\ tan\left(\frac{1}{2}\ x\ +\ \frac{1}{4}\ \pi\right)\right|$	$sec\ x$	*
$-\ ln\ \left|\ csc\ x\ +\ cot\ x\ \right|$	$csc\ x$	*
$ln\ \left|\ tan\left(\frac{1}{2}\ x\right)\right|$	$csc\ x$	*
e^x	e^x	

As has been seen, many questions require the application of a trigonometric identity, but the useful identities are not given explicitly.

The ***three key*** identities should be memorised;

$$cos^2\ \theta\ +\ sin^2\ \theta\ =\ 1$$

$$cos^2\ \theta\ -\ sin^2\ \theta\ =\ cos\ 2\theta$$

$$2\ sin\ \theta\ cos\ \theta\ =\ sin\ 2\theta$$

From the ***three key***, the ***following four*** are easily obtained;

$$1\ +\ tan^2\ \theta\ =\ sec^2\ \theta$$

$$cot^2\ \theta\ +\ 1\ =\ csc^2\ \theta$$

$$2\ cos^2\ \theta\ =\ 1\ +\ cos\ 2\theta \qquad \text{Essential to find } \int cos^2\ \theta\ d\theta$$

$$2\ sin^2\ \theta\ =\ 1\ -\ cos\ 2\theta \qquad \text{Essential to find } \int sin^2\ \theta\ d\theta$$

Either memorise the ***following four***, or learn how to obtain them from the ***three key***.

5.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 28

When trigonometry and calculus mix, **RADIANS MUST BE USED !**

Question 1
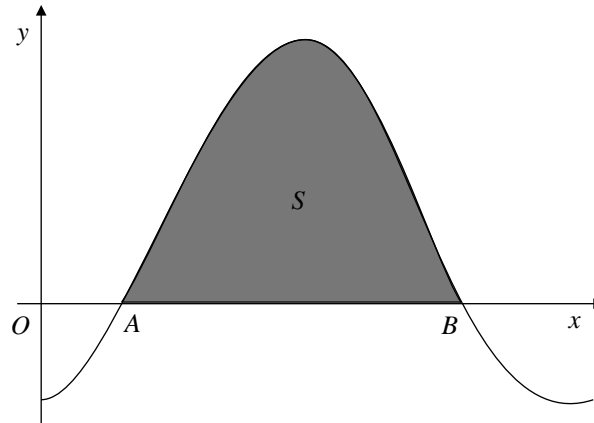*A-Level Examination Question from January 2010, Paper C4, Q8 (a) (Edexcel)*
Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_{1}^{\sqrt{2}} \frac{1}{x^2 \sqrt{4 - x^2}} \, dx$$

[7 marks]

Question 2

A-Level Examination Question from January 2013, Paper C4, Q6 (a) (Edexcel)

Shown is a sketch of the curve with equation $y = 1 - 2\cos x$, where x is measured in radians. The curve crosses the x-axis at the point A and the point B.
Find, in terms of π, the x coordinate of the point A and the x coordinate of the point B.

[3 marks]

Question 3

A-Level Examination Question from June 2013, Paper C4, Q5 (Edexcel)

(a) Use the substitution $x = u^2$, $u > 0$, to show that,

$$\int \frac{1}{x \left(2 \sqrt{x} - 1 \right)} \, dx = \int \frac{2}{u \left(2u - 1 \right)} \, du$$

[3 marks]

(b) Hence show that

$$\int_1^9 \frac{1}{x \left(2 \sqrt{x} - 1 \right)} \, dx = 2 \, ln \left(\frac{a}{b} \right)$$

where a and b are integers to be determined.

[7 marks]

Question 4

A-Level Examination Question from June 2004, Paper P3, Q4 (Edexcel)

Use the substitution $u = 1 + \sin x$ and integration to show that

$$\int \sin x \cos x \, (1 + \sin x)^5 \, dx = \frac{1}{42}(1 + \sin x)^6 \, (6 \sin x - 1) + constant$$

[8 marks]