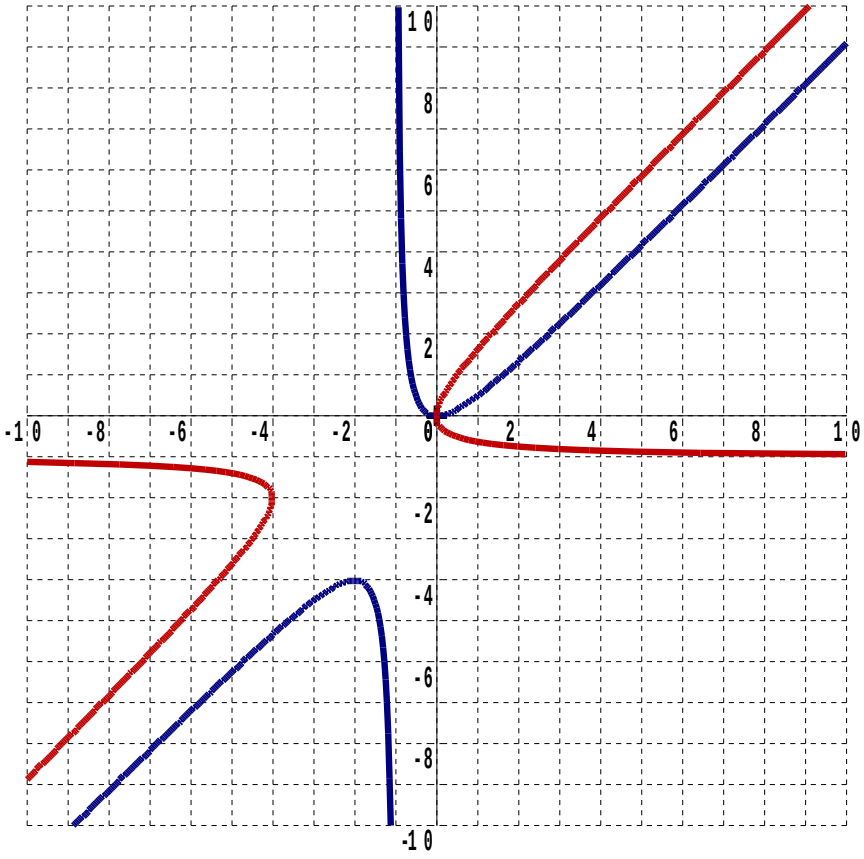
A-Level Pure Mathematics

Year 1
Graph Work
featuring
Graph Transformations

G • R • A • P • H W • O• R • K



The graphs of $y = \frac{x^2}{x+1}$ (in blue) and $x = \frac{y^2}{y+1}$ (in red)

Each is a reflection of the other in the line $y = x$

Lesson 1

A-Level Pure Mathematics : Year 1
Graphwork

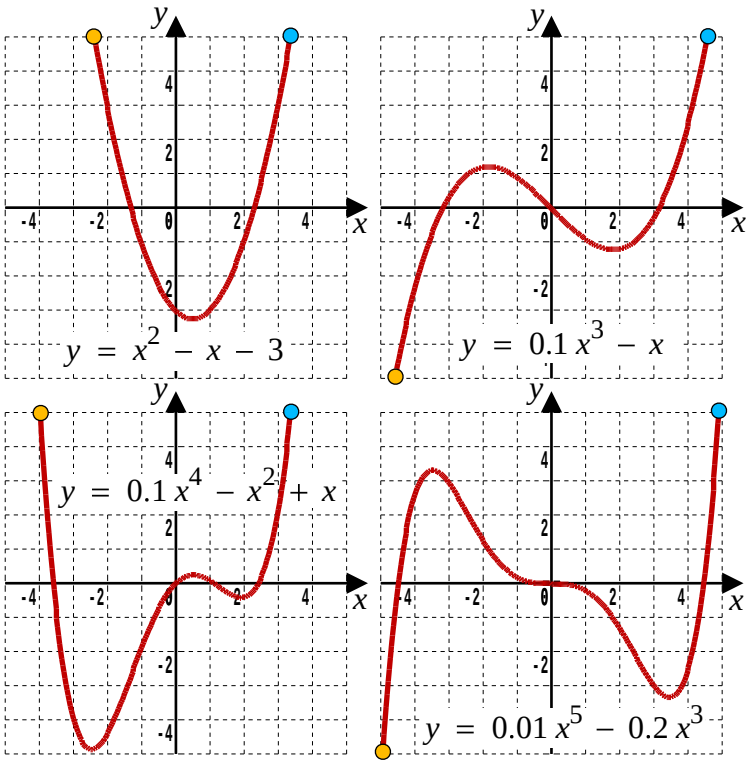
1.1 Graphing Polynomials

Faced with the task of quickly sketching a polynomial how best to proceed ?
It turns out that the degree of the polynomial is of crucial importance.

The Degree of a Polynomial
The degree of a polynomial the highest power of *x*.

1.2 Graphed Polynomials with Positive Highest Order Term

Here are a few examples of polynomials and their corresponding graphs.
For each, note the parity (odd or even) of the degree of the polynomial and, for large values of *x*, which quadrant the graph enters and exits.
(Remember that graphs are always read, like this sentence, from left to right)



Here is a summary of the key observations to be made,

Equation	Degree	Degree's Parity	Entry	Exit
$y = x^2 - x - 3$	2	even	2nd	1st
$y = 0.1\,x^3 - x$	3	odd	3rd	1st
$y = 0.1\,x^4 - x^2 + x$	4	even	2nd	1st
$y = 0.01\,x^5 - 0.2\,x^3$	5	odd	3rd	1st

The observations made suggest the first half of the following rule.

Large Values of x Rule for Polynomials

If the highest order term is positive then the exit is in the 1st quadrant.

Additionally,

- if the degree is even the entry is in the 2nd quadrant.
- if the degree is odd the entry is in the 3rd quadrant.

If the highest order term is negative then the exit is in the 4th quadrant.

Additionally,

- if the degree is even the entry is in the 3rd quadrant.
- if the degree is odd the entry is in the 2nd quadrant

1.3 Example

$$f(x) = x^4 + 4x^3 - 17x^2 - 60x$$

Given that $(x - 4)$ is a factor, factorise $f(x)$ completely and hence sketch the graph of $f(x)$ clearly marking all axis crossing points.

[8 marks]

1.4 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 74

Question 1

$$f(x) = x^4 + 2x^3 - 11x^2 - 12x$$

Given that $(x - 3)$ is a factor, factorise $f(x)$ completely and hence sketch the graph of $f(x)$ clearly marking all axis crossing points.

[8 marks]

Question 2

$$f(x) = 6x^3 + 7x^2 - 23x - 30$$

(i) Calculate the value of $f(1)$, the value of $f(2)$, and the value of $f(3)$

[3 marks]

The Factor Theorem

If, for a given polynomial function $p(x)$, $p(a) = 0$ (for some constant, a)

then $(x - a)$ is a factor of $p(x)$

(ii) Hence, using the factor theorem, factorise $f(x)$ completely and so sketch the graph of $f(x)$ clearly marking all axis crossing points.

[8 marks]

Question 3

$$p(x) = a^2 - x^2 \qquad \text{where } a \text{ is a positive constant}$$

Sketch the graph of $p(x)$ clearly making all axis crossing points in terms of a.

[3 marks]

Question 4

$$q(x) = x^2 + a^2 \qquad \text{where } a \text{ is a positive constant}$$

Sketch the graph of $q(x)$ clearly making all axis crossing points in terms of a.

[3 marks]

Question 5

$$r(x) = x^4 - a^4 \quad \text{where } a \text{ is a positive constant}$$

Sketch the graph of $r(x)$ clearly making all axis crossing points in terms of a.

[3 marks]

Question 6

$$f(x) = 7x^3 - 36x - x^4$$

Given that $(x + 2)$ is a factor, factorise $f(x)$ completely and hence sketch the graph of $f(x)$ clearly marking all axis crossing points.

[8 marks]

Question 7

A-Level Examination Question from June 2019, Paper 1, Q1 (Edexcel)

$$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$ find the value of the constant a

[3 marks]

Question 8

Continue the previous question by factorising $f(x)$ completely, using the value of a found, and so sketch the graph of $f(x)$ marking on its two axis crossing points.

[8 marks]

Question 9

Given that $f(x) = (x - 10)(x^2 - 2x) + 12x$

(i) Express $f(x)$ in the form $x(ax^2 + bx + c)$ where a, b and c are real constants

[3 marks]

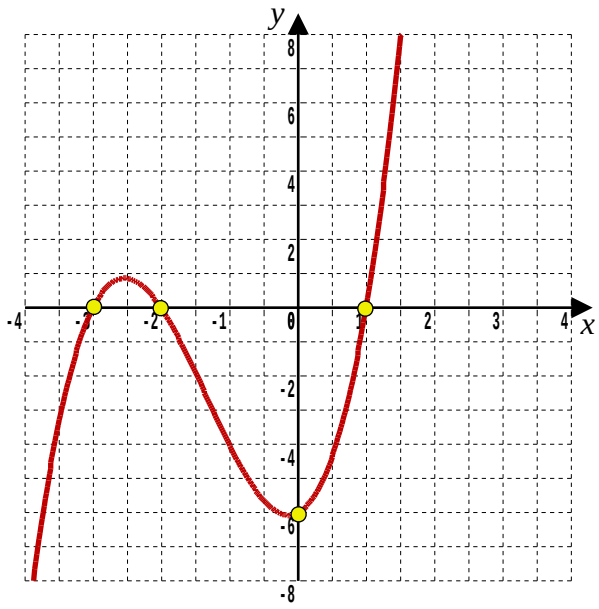
(ii) Hence factorise $f(x)$ completely.

[2 marks]

(iii) Sketch the graph of $y = f(x)$ showing clearly the points where the graph intersects the axes.

[3 marks]

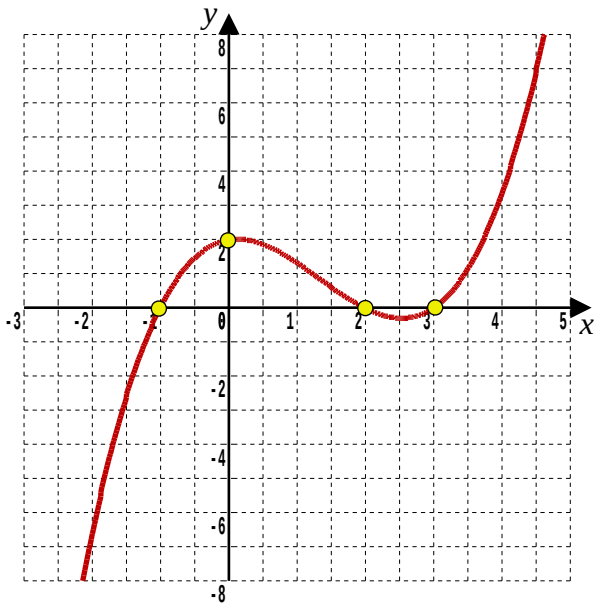
Question 10



The graph is of $y = x^3 + bx^2 + cx + d$, where b, c and d are real constants.
Find the values of b, c and d.

[3 marks]

Question 11



The graph is of $y = ax^3 + bx^2 + cx + d$, where a, b, c and d are real constants.
Find the values of a, b, c and d.

[4 marks]

Question 12

$$f(x) = 16 - 9x^2 - 6x^3 - x^4$$

(i) Given that the roots of $f(x)$ have integer values, list the ten possible roots suggested by the term independent of x being 16.

[2 marks]

(ii) Use the factor theorem to find two roots of $f(x)$

[2 marks]

(iii) Hence, factorise $f(x)$ completely and so sketch the graph of $f(x)$ clearly marking all axis crossing points.

[8 marks]