Year FM Further Pure Mathematics Examination Revision : Health Check N° 4

I told the Doctor I didn't want brain surgery.
He changed my mind !

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 32

Question 1

Find the value of x for which

$$2\,tanh\,x - 1 = 0$$

giving your answer in terms of a natural logarithm.

[4 marks]

Question 2

$$y = \sin x \cosh x$$

(i) Show that $\dfrac{d^4y}{dx^4} = -4y$

[4 marks]

(ii) Hence find the first three non-zero terms of the Maclaurin series for *y*.
Give each coefficient in its simplest form.

[4 marks]

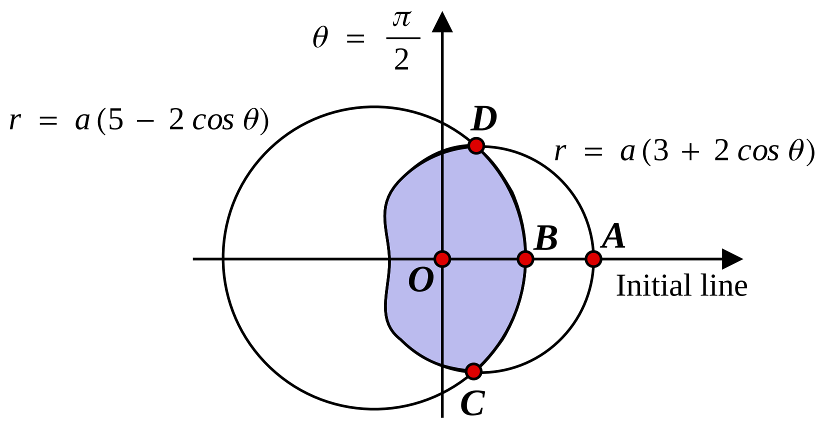
Question 3

A logo is designed which consists of two overlapping closed curves.
The polar equations of these curves are,

$$r = a(3 + 2\cos\theta), \quad 0 \leqslant \theta \leqslant 2\pi$$

$$r = a(5 - 2\cos\theta), \quad 0 \leqslant \theta \leqslant 2\pi$$

Given below is a sketch (not to scale) of these two curves.



(i) Write down the polar coordinates of the points A and B where the curves meet the initial line,

[2 marks]

(ii) Find the polar coordinates of the points C and D where the curves meet.

[4 marks]

(iii) Show that the area of the overlapping region, which is shaded in the diagram, is $\dfrac{a^2}{3}\left(49\pi - 48\sqrt{3}\right)$

[6 marks]

Question 4

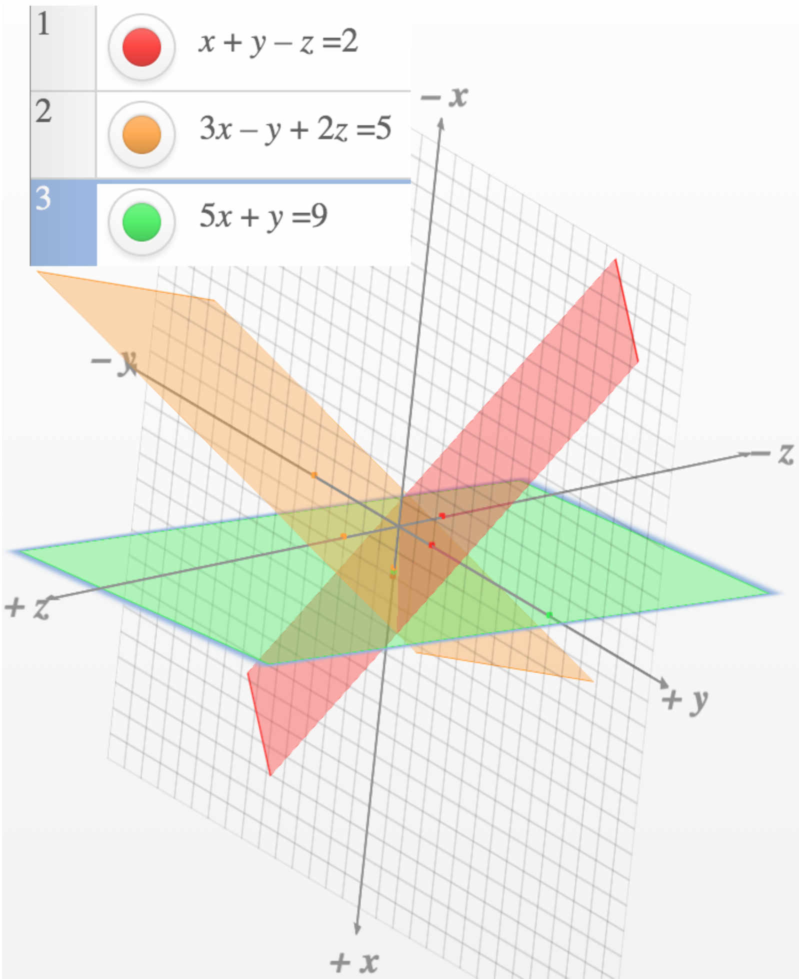
A three dimensional graph plotter is used to plot the planes with equations,

$$x + y - z = 2 \qquad \text{Equation 1}$$

$$3x - y + 2z = 5 \qquad \text{Equation 2}$$

$$5x + y = 9 \qquad \text{Equation 3}$$

From the plot it looks as if, rather than intersecting at a common point, all three planes intersect along a line. This configuration of planes is called a sheaf.



(i) Consider the three planes written in matrix form,

$$\begin{pmatrix} 1 & 1 & -1 \\ 3 & -1 & 2 \\ 5 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 9 \end{pmatrix}$$

Show that the 3×3 matrix in this equation has a determinant of zero.

[2 marks]

(ii) A determinant of zero means no inverse to the 3×3 matrix exists,
The matrix equation cannot be solved thus confirming that there is no unique point of intersection of all three planes.

The original equations were labelled Equation 1, Equation 2 and Equation 3. Show that, if z is eliminated by combining Equation 1 and Equation 2, then Equation 3 is obtained.

This shows that all three planes intersect in the line.

[2 marks]

(iii) Show that the points (0, 9, 7) and (1, 4, 3) are on all three planes and hence obtain the vector equation of the line of intersection in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$

[4 marks]