Year FM Further Pure Mathematics Examination Revision : Health Check N° 3

The Doctor will see you now

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 30

Question 1

By considering $\left(z + \frac{1}{z} \right)^3$ where $z = \cos\theta + \mathrm{i}\sin\theta$, show that,

$$\cos^3\theta = \frac{1}{4}\left(\cos 3\theta + 3\cos\theta \right)$$

[4 marks]

Question 2

(i) Use the substitution $x = a \tan\theta$ to show that,

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + c$$

(This "quotable without proof" result is given in the examination formula booklet, but, as here, you may be asked to prove it)

[3 marks]

(ii) Given that $f(x) = \dfrac{8x - 3}{4 + x^2}$ find $\int f(x)\, dx$, presenting your answer in the form $A \ln\left(x^2 + 4\right) + B \arctan\left(\dfrac{x}{2}\right) + c$ where c is an arbitrary constant and A and B are constants to be found.

[4 marks]

Question 3

$\mathbf{A} = \begin{pmatrix} 2p & p & 2 \\ 3 & 0 & 0 \\ -1 & 1 & -1 \end{pmatrix}$ where p is a real constant.

Given that $\mathbf{A}$ is non-singular, find $\mathbf{A}^{-1}$ in terms of p.

[4 marks]

Question 4

Prove by induction that for all positive integers n, $2^{6n} + 3^{2n-2}$ is divisible by 5

[6 marks]

Question 5

The line l_1 has equation $\frac{x-2}{2} = \frac{y-4}{(-2)} = \frac{z+6}{1}$

The plane Π has equation $2x - 3y + z = 8$

The line l_2 is the reflection of line l_1 in the plane Π.

Find a vector equation of the line l_2

[9 marks]