



The Doctor will see you now

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 30

Question 1

By considering $\left(z + \frac{1}{z}\right)^3$ where $z = \cos \theta + i \sin \theta$, show that,

$$\cos^3 \theta = \frac{1}{4} (\cos 3\theta + 3 \cos \theta)$$

[4 marks]

Question 2

- (i) Use the substitution $x = a \tan \theta$ to show that,

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + c$$

(This “quotable without proof” result is given in the examination formula booklet, but, as here, you may be asked to prove it)

[3 marks]

- (ii) Given that $f(x) = \frac{8x - 3}{4 + x^2}$ find $\int f(x) dx$, presenting your answer in the form $A \ln(x^2 + 4) + B \arctan\left(\frac{x}{2}\right) + c$ where c is an arbitrary constant and A and B are constants to be found.

[4 marks]

Question 3

$$\mathbf{A} = \begin{pmatrix} 2p & p & 2 \\ 3 & 0 & 0 \\ -1 & 1 & -1 \end{pmatrix} \text{ where } p \text{ is a real constant.}$$

Given that \mathbf{A} is non-singular, find \mathbf{A}^{-1} in terms of p .

[4 marks]

Question 4

Prove by induction that for all positive integers n , $2^{6n} + 3^{2n-2}$ is divisible by 5

[6 marks]

Question 5

The line l_1 has equation $\frac{x - 2}{2} = \frac{y - 4}{(-2)} = \frac{z + 6}{1}$

The plane Π has equation $2x - 3y + z = 8$

The line l_2 is the reflection of line l_1 in the plane Π .

Find a vector equation of the line l_2

[9 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk