



Why did the mattress go to the doctor ?  
It had spring fever

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 25

**Question 1**

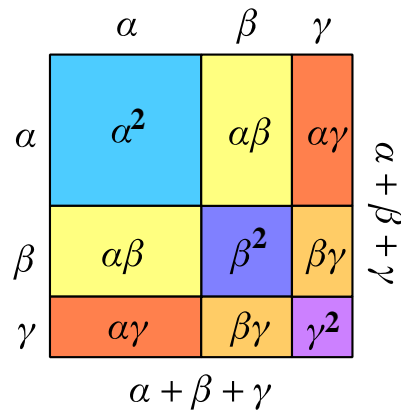
- ( i ) A famous trigonometric identity is that,  $\sin 3x = 3 \sin x - 4 \sin^3 x$   
By use of Osborn's Rule, or otherwise, write down the similar identity  
for the hyperbolic function  $\sinh 3x$

[ 1 mark ]

- ( ii ) Using the definition of  $\sinh x$  in terms of exponentials prove that your  
part (i) identity is correct.

[ 3 marks ]

**Question 2**



(i) With the aid of the diagram expand the brackets of  $(\alpha + \beta + \gamma)^2$

[ 1 mark ]

The roots of the equation  $4x^3 - 12x^2 - x + 3 = 0$  are  $\alpha, \beta$  and  $\gamma$   
Without solving the equation, write down the values of,

(ii)  $\alpha + \beta + \gamma$

[ 1 mark ]

(iii)  $\alpha\beta + \beta\gamma + \gamma\alpha$

[ 1 mark ]

(iv)  $\alpha\beta\gamma$

[ 1 mark ]

(v)  $\alpha^2 + \beta^2 + \gamma^2$

[ 3 marks ]

### Question 3

Javier is revising Volumes of Revolution, and reads in his notes that;

---

#### Parametric Volumes of Revolution

The volume of revolution formed when the parametric curve with equations

$$x = f(\theta) \quad \text{and} \quad y = g(\theta)$$

is rotated through  $2\pi$  radians about the  $x$ -axis between  $x = a$  and  $x = b$  is,

$$V = \pi \int_{x=a}^{x=b} y^2 dx = \pi \int_{\theta=q}^{\theta=p} y^2 \frac{dx}{d\theta} d\theta$$

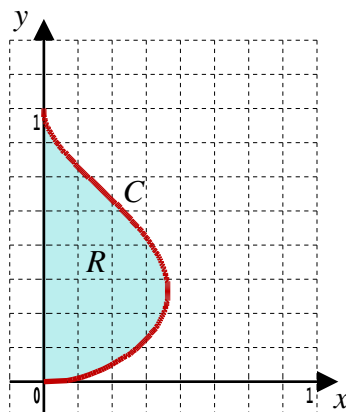
The same curve rotated  $2\pi$  radians about the  $y$ -axis between  $y = a$  and  $y = b$  is,

$$V = \pi \int_{y=a}^{y=b} x^2 dy = \pi \int_{\theta=q}^{\theta=p} x^2 \frac{dy}{d\theta} d\theta$$

---

The curve  $C$ , graphed below, has the parametric equations,

$$x = \sin^4 \theta \sqrt{\cos \theta}, \quad y = \cos \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$



The finite region  $R$  bounded by the curve and the  $y$ -axis is rotated through  $2\pi$  radians about the  $y$ -axis. Find the volume of the solid of revolution formed.

[ 4 marks ]

**Question 4**

- (a) Use the standard results for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$  to show that,

$$\sum_{r=1}^n (r^2 - r - 1) = \frac{1}{3} n (n - 2) (n + 2)$$

for all positive integers  $n$

[ 4 marks ]

- (b) Hence calculate  $\sum_{r=10}^{40} (r^2 - r - 1)$

[ 3 marks ]

**Question 5**

*Further AS-Level Examination Question from June 2018, Paper 1, Q16 (AQA)*

Two matrices **A** and **B** satisfy the equation  $\mathbf{AB} = \mathbf{I} + 2\mathbf{A}$  where **I** is the identity

matrix and  $\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -4 & 8 \end{pmatrix}$

Find **A**

**[ 3 marks ]**

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School

It may be freely duplicated and distributed, unaltered, for non-profit educational use

In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

© 2023 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from [mhh@shrewsbury.org.uk](mailto:mhh@shrewsbury.org.uk)