Why did the mattress go to the doctor ?
It had spring fever

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 25

Question 1

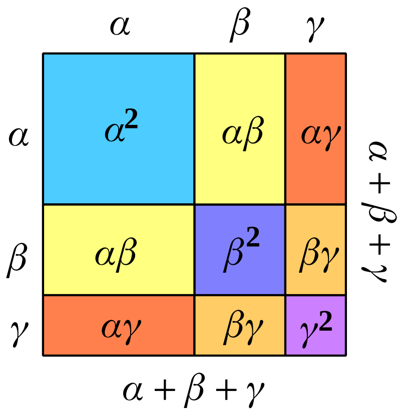
(i) A famous trigonometric identity is that, $\sin 3x = 3\sin x - 4\sin^3 x$
By use of Osborn's Rule, or otherwise, write down the similar identity for the hyperbolic function $\sinh 3x$

[1 mark]

(ii) Using the definition of $\sinh x$ in terms of exponentials prove that your part (i) identity is correct.

[3 marks]

Question 2



(i) With the aid of the diagram expand the brackets of $(\alpha + \beta + \gamma)^2$

[1 mark]

The roots of the equation $4x^3 - 12x^2 - x + 3 = 0$ are α, β and γ

Without solving the equation, write down the values of,

(ii) $\alpha + \beta + \gamma$

[1 mark]

(iii) $\alpha\beta + \beta\gamma + \gamma\alpha$

[1 mark]

(iv) $\alpha\beta\gamma$

[1 mark]

(v) $\alpha^2 + \beta^2 + \gamma^2$

[3 marks]

Question 3
Javier is revising Volumes of Revolution, and reads in his notes that;

Parametric Volumes of Revolution
The volume of revolution formed when the parametric curve with equations

$$x = f(\theta) \quad \text{and} \quad y = g(\theta)$$

is rotated through 2π radians about the x-axis between $x = a$ and $x = b$ is,

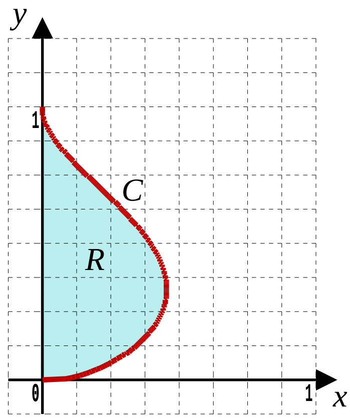
$$V = \pi \int_{x=a}^{x=b} y^2 \, dx = \pi \int_{\theta=q}^{\theta=p} y^2 \frac{dx}{d\theta} \, d\theta$$

The same curve rotated 2π radians about the y-axis between $y = a$ and $y = b$ is,

$$V = \pi \int_{y=a}^{y=b} x^2 \, dy = \pi \int_{\theta=q}^{\theta=p} x^2 \frac{dy}{d\theta} \, d\theta$$

The curve C, graphed below, has the parametric equations,

$$x = \sin^4\theta \sqrt{\cos\theta}, \qquad y = \cos\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$



The finite region R bounded by the curve and the y-axis is rotated through 2π radians about the y-axis. Find the volume of the solid of revolution formed.

[4 marks]

Question 4

(a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that,

$$\sum_{r=1}^{n} \left(r^2 - r - 1 \right) = \frac{1}{3} n (n - 2) (n + 2)$$

for all positive integers *n*

[4 marks]

(b) Hence calculate $\sum_{r=10}^{40} \left(r^2 - r - 1 \right)$

[3 marks]

Question 5

Further AS-Level Examination Question from June 2018, Paper 1, Q16 (AQA)

Two matrices **A** and **B** satisfy the equation $\mathbf{AB} = \mathbf{I} + 2\mathbf{A}$ where **I** is the identity matrix and $\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -4 & 8 \end{pmatrix}$

Find **A**

[3 marks]