10.1 By Parts and Substitution

This lesson is a revision of the two topics Integration by Parts and Integration by Substitution but with an emphasis on situations that involve the hyperbolic functions and their associated trigonometric identities.

Keep in mind that carrying out an integration using parts (LIDI) or a substitution can be a lot of work and there is a definite time saving in opting for a "chain rule backwards" if a question is amenable to that approach. However, an examination question may prescribe that the solution method has to be "by parts" or "by substitution", or the parts or substitution methods may genuinely be the easier method of doing the question.

10.2 Example #1 (By Parts)

Use integration by parts to find $\int x \sinh 3x \, dx$

[3 marks]

10.3 Example #2 (Trigonometric Odd Powers)

In general to integrate a trigonometric function raised to an odd power the following simple manoeuvre can be very effective.

Trigonometric Odd Powers Integration

For small odd values of n, use the cunning moves,

$$\int \sinh^n x \, dx = \int \sinh x \sinh^{n-1} x \, dx$$

$$\int \cosh^n x \, dx = \int \cosh x \cosh^{n-1} x \, dx$$

Find $\int \sinh^3 x \, dx$

[3 marks]

10.4 Example #3 (Substitution)

When carrying out an integration involving a substitution $x = f(u)$ remember that three items need attending to:

- Making the substitution into the function being integrated
- Replacing the dx with du having first worked out $\frac{dx}{du}$
- Replacing the "x =" limits with "u =" limits

Use the substitution $x = 3\,sinh\,u$ to find the exact value of $\int_0^6 \frac{x^3}{\sqrt{x^2 + 9}}\,dx$

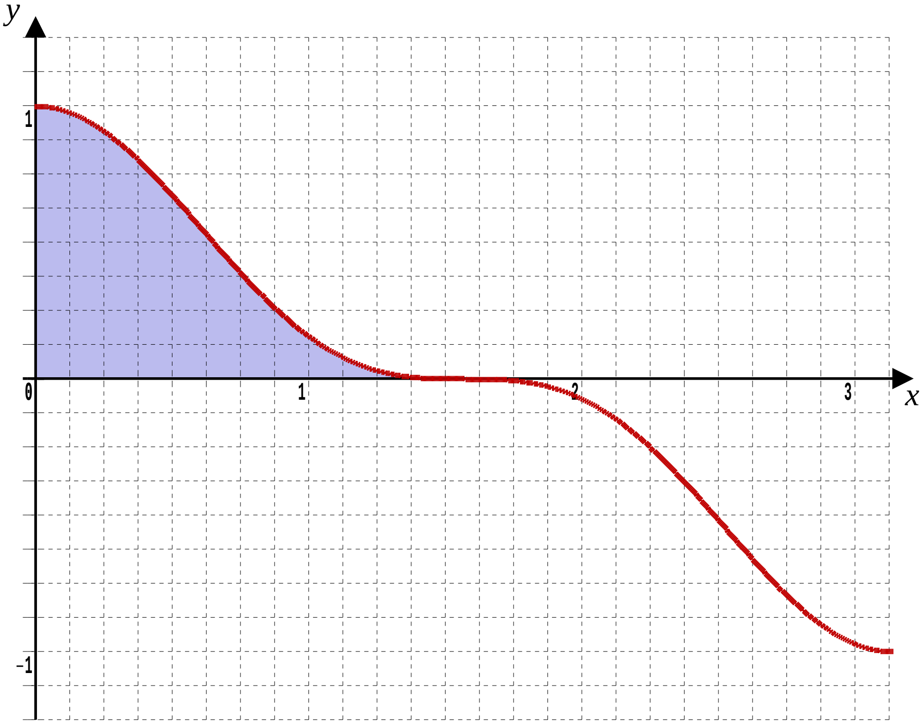
[11 marks]

10.5 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 30

Question 1

The graph is of the function $f\,(x) \;=\; cos^3 x, \quad 0 \leqslant x \leqslant \pi$



Find the area shaded which is bounded by $f\,(x)$ and the positive coordinate axes.

[5 marks]

Question 2

Use integration by parts to find $\int 25x^2 \cosh(5x)\ dx$

[5 marks]

Question 3

Find $\int \cosh^3 x\, dx$

[3 marks]

Question 4

Use the substitution $x = 6 \cosh u$ to find the exact value of $\int_{12}^{18} \frac{x^3}{\sqrt{x^2 - 36}}\, dx$

Give your answer in the form $s\left(p\sqrt{2} - q\sqrt{3} \right)$ where s is a square number and p and q are prime numbers.

[11 marks]

Question 5

Using the substitution $x = sec\,\theta$ find,

(i) $\displaystyle\int \frac{1}{x\sqrt{x^2 - 1}}\,dx$

[2 marks]

(ii) $\displaystyle\int \frac{\sqrt{x^2 - 1}}{x}\,dx$

[4 marks]