Lesson 4

Further A-Level Pure Mathematics, Core 2

Maclaurin Series

4.1 The Inverse Trig Functions

A few derivatives of inverse trigonometric functions are amongst those given in the examination formula booklet. Even so, an examination may well ask for one of the results given to be proven.

$f(x)$	$f'(x)$	In Formula Book ?
x^n	$n\,x^{n-1}$	No
e^x	e^x	No
$\ln x$	$\frac{1}{x}$	No
$\sin x$	$\cos x$	No
$\cos x$	$-\sin x$	No
$\tan x$	$\sec^2 x$	Yes
$\csc x$	$-\csc x \cot x$	Yes
$\sec x$	$\sec x \tan x$	Yes
$\cot x$	$-\csc^2 x$	Yes
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	Yes
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$	Yes
$\arctan x$	$\frac{1}{1+x^2}$	Yes

Example #1

Prove that the derivative of $\arcsin x$ is $\dfrac{1}{\sqrt{1-x^2}}$

Teaching Video : http://www.NumberWonder.co.uk/v9028/10b.mp4

[5 marks]

4.2 The Binomial Theorem

To find the Maclaurin series for *arcsin x* use could be made of the rule,

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The Generalised Maclaurin Series

A given function, $f(x)$, may be written as the polynomial,

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(r)}(0)}{r!}x^r + \ldots$$

provided that $f(0)$, $f'(0)$, $f''(0)$, ..., $f^{(r)}(0)$, ... all have finite values

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However, the Binomial Theorem provides an easier way.

Example #2

With the help of the Binomial Theorem, find the first four non-zero terms in the Maclaurin series for *arcsin x*

Teaching Video : http://www.NumberWonder.co.uk/v9098/4.mp4

[8 marks]

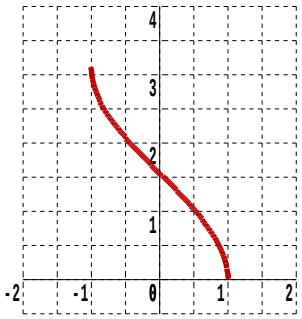
4.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 40

Question 1

With the aid of the graph, prove that the derivative of *arccos x* is $-\dfrac{1}{\sqrt{1-x^2}}$



$y = \arccos x$

[5 marks]

Question 2

(i) It is obviously true that,

$$\frac{1}{\sqrt{1-x^2}} + \left(-\frac{1}{\sqrt{1-x^2}}\right) = 0$$

By integrating each term in this equation, deduce a simple relationship between *arcsin x* and *arccos x*

[3 marks]

(ii) Hence, use the Maclaurin series for *arcsin x* to deduce the Maclaurin series for *arccos x.*

[2 marks]

Question 3

Prove that the derivative of *arctan x* is $\frac{1}{1 + x^2}$

[5 marks]

Question 4

With the help of the Binomial Theorem, find the first four non-zero terms in the Maclaurin series for *arctan x*

[8 marks]

Question 5

Prove that the derivative of *arccot x* is $-\dfrac{1}{1 + x^2}$

[5 marks]

Question 6

(i) It is obviously true that,

$$\frac{1}{1 + x^2} + \left(-\frac{1}{1 + x^2}\right) = 0$$

By integrating each term in this equation, deduce a simple relationship between *arctan x* and *arccot x*

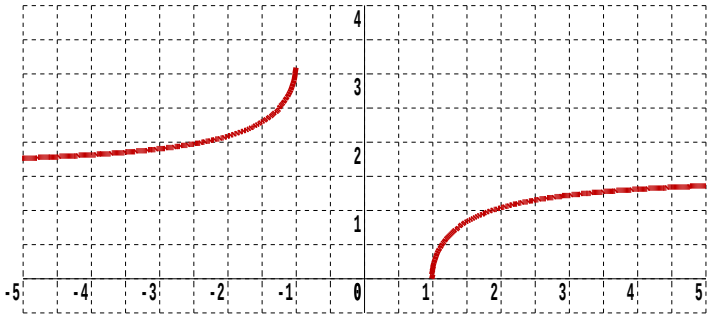
[3 marks]

(ii) Hence, use the Maclaurin series for *arctan x* to deduce the Maclaurin series for *arccot x.*

[2 marks]

Question 7

With the aid of the graph, prove that the derivative of *arcsec x* is $\frac{1}{x\sqrt{x^2 - 1}}$



The graph of $y = arcsec\, x$

[5 marks]

Question 8

Give a reason why a Maclaurin series for *arcsec x* will not exist.

[2 marks]