Lesson 2

2.1 Streamlining

If the process of finding the best polynomial of degree n to a given function is streamlined, the resulting general rule is termed the Maclaurin Series.
It's traditionally written in function notation rather than Leibniz's notation.

The Generalised Maclaurin Series

A given function, $f(x)$, may be written as the polynomial,

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(r)}(0)}{r!}x^r + \ldots$$

provided that $f(0)$, $f'(0)$, $f''(0)$, ..., $f^{(r)}(0)$, ... all have finite values

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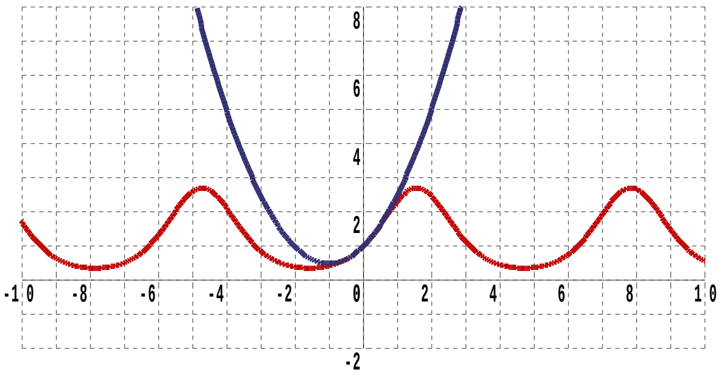
2.2 Example

Find the first three terms of the Maclaurin series for $f(x) = e^{\sin x}$



Teaching Video : http://www.NumberWonder.co.uk/v9098/2.mp4

[6 marks]

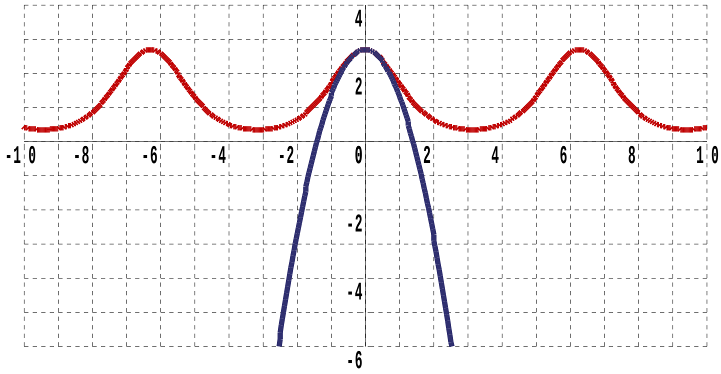
2.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 50

Question 1

Find the first three terms of the Maclaurin series for $f(x) = e^{\cos x}$

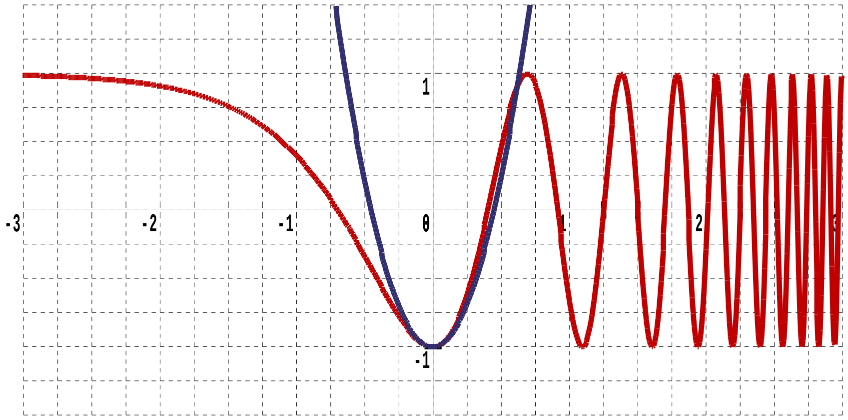


In red is the graph of $y = e^{\cos x}$ and in blue the best quadratic approximation centred on $x = 0$

[6 marks]

Question 2

Find the first three terms of the Maclaurin series for $f(x) = \cos(\pi e^{x})$

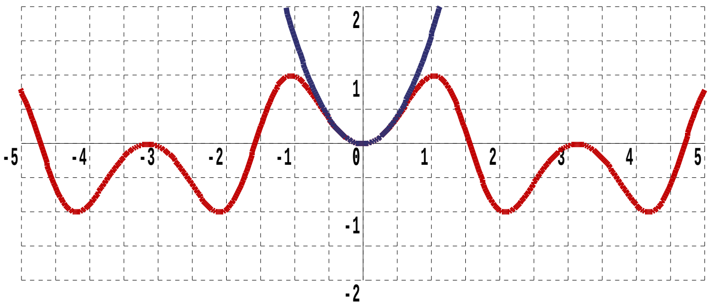


In red is the graph of $y = \cos(\pi e^{x})$ and in blue the best quadratic approximation centred on $x = 0$

[6 marks]

Question 3

Find the first three terms of the Maclaurin series for $f(x) = sin(\pi \ cos \ x)$

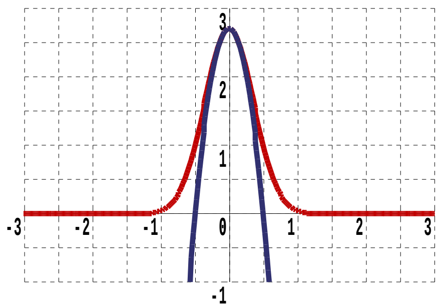


In red is the graph of $y = sin(\pi \, cos \, x)$ and in blue the best quadratic approximation centred on $x = 0$

[6 marks]

Question 4

Find the first three terms of the Maclaurin series for $f(x) = e^{1-x^2}$



In red is the graph of $y = e^{1-x^2}$ and in blue the best quadratic approximation centred on $x = 0$

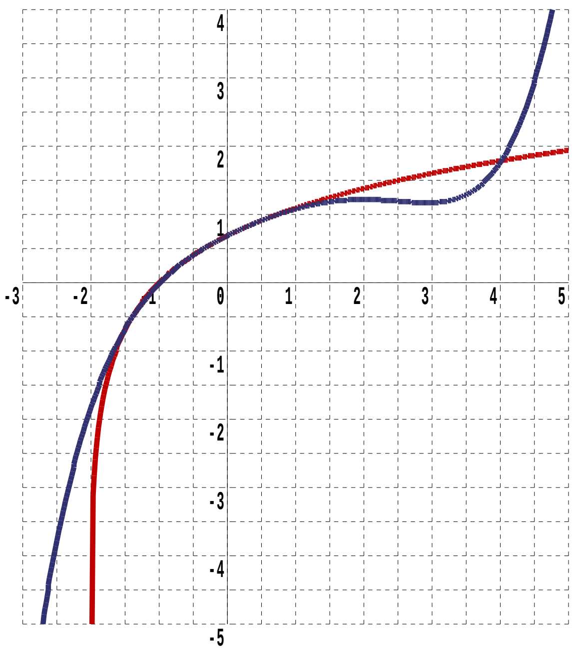
[6 marks]

Question 5

(i) Complete the table to show the first, second, third, fourth and fifth derivatives of the function $y = ln(2 + x)$ where $x > -2$
Also work out values of those derivatives at $x = 0$

$f(x) = ln(2 + x)$	when $x = 0,$	$f(0) =$
$f'(x) =$	when $x = 0,$	$f'(0) =$
$f''(x) =$	when $x = 0,$	$f''(0) =$
$f'''(x) =$	when $x = 0,$	$f'''(0) =$
$f^{(4)}(x) =$	when $x = 0,$	$f^{(4)}(0) =$
$f^{(5)}(x) =$	when $x = 0,$	$f^{(5)}(0) =$

(ii) Hence determine the best quintic polynomial approximation to the function $y = ln(2 + x)$ for values of x that are close to zero.

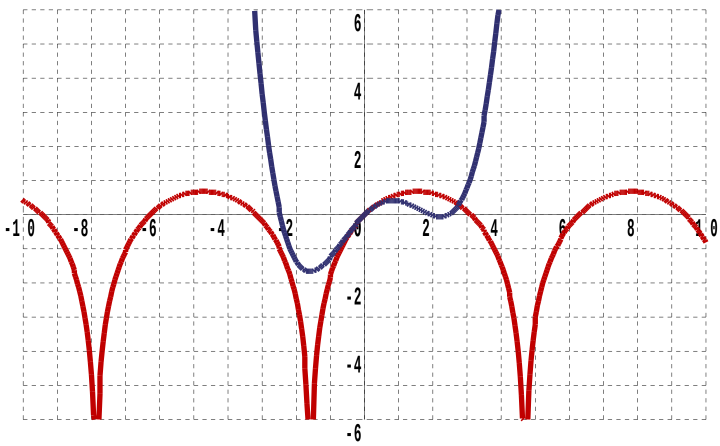


In red is the graph of $y = ln(2 + x)$ and in blue the best quintic polynomial approximation on $x = 0$

[8 marks]

Question 6

Showing full working, determine the best quartic polynomial approximation to the function $y = ln(1 + sin x)$ for values of x that are close to zero.



In red is the graph of $y = ln(1 + sin x)$ and in blue the best quartic polynomial approximation on $x = 0$

[10 marks]