

Lesson 2

Further A-Level Pure Mathematics : Core 1 Complex Numbers I

2.1 Complex Number Division

A complex number, z , is a number of the form,

$$z = a + bi$$

where a and b are real numbers, $i = \sqrt{-1}$ and $i^2 = -1$

Given two complex numbers,

$$z = a + bi \quad \text{and} \quad w = c + di$$

the arithmetic operations of addition, subtraction and multiplication can be performed with the two numbers and the result is in the form of a complex number.

Addition :

$$\begin{aligned} z + w &= (a + bi) + (c + di) \\ &= (a + c) + (b + d)i \end{aligned}$$

As a, b, c and d are all real numbers, $(a + c)$ and $(b + d)$ will also be real numbers which we could call u and v .

$$\therefore z + w = u + vi \quad \text{which is in the form of a complex number}$$

Subtraction :

$$\begin{aligned} z - w &= (a + bi) - (c + di) \\ &= (a - c) + (b - d)i \end{aligned}$$

As a, b, c and d are all real numbers, $(a - c)$ and $(b - d)$ will also be real numbers which we could call u and v .

$$\therefore z - w = u + vi \quad \text{which is in the form of a complex number}$$

Multiplication :

$$\begin{aligned} zw &= (a + bi)(c + di) \\ &= ac + adi + bci + bdi^2 \\ &= ac + adi + bci + bd(-1) \\ &= (ac - bd) + (ad + bc)i \end{aligned}$$

As a, b, c and d are all real numbers, $(ac - bd)$ and $(ad + bc)$ will also be real numbers which we could call u and v .

$$\therefore zw = u + vi \quad \text{which is in the form of a complex number}$$

This all leads to an obvious question; "What about division ?".

Can any complex number, divided by any other (non zero) complex number be manipulated into the form of a complex number ?

Teaching Video : [http://www.NumberWonder.co.uk/Video/v9085\(2\).mp4](http://www.NumberWonder.co.uk/Video/v9085(2).mp4)



Division : $\frac{z}{w} = \frac{a + bi}{c + di}$

[3 marks]

2.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 45

Question 1

Three complex numbers are,

$$u = 5 - 3i \quad v = 4 + 2i \quad \text{and} \quad w = \frac{u}{v}$$

- (i) Write down v^* the complex conjugate of v .

[1 mark]

- (ii) Show how to use v^* to write w in the form $a + bi$ where $a, b \in \mathbb{Q}$

[3 marks]

Question 2

Given that $u = 3 + i$ write $\frac{u}{u^*}$ in the form $a + bi$ where $a, b \in \mathbb{Q}$

[3 marks]

Question 3

Given that,

$$z_1 = 1 + i \quad z_2 = 2 + i \quad \text{and} \quad z_3 = 3 + i$$

write each of the following in the form $a + bi$ where $a, b \in \mathbb{Q}$

(i) $\frac{z_1 z_2}{z_3}$

[3 marks]

(ii) $\frac{(z_2)^2}{z_1}$

[3 marks]

(iii) $\frac{2z_1 + 5z_3}{z_2}$

[3 marks]

Question 4

Simplify, $\frac{2 + i}{2 - i} - \frac{2 - i}{2 + i}$

[4 marks]

Question 5

Given that

$$w = \sqrt{3} + \sqrt{2} i$$

write w^{-1} in the form $a + bi$ where $a, b \in \mathbb{R}$

[4 marks]

Question 6

Simplify, $\frac{1}{i} + \frac{1}{1+i} + \frac{1}{1-i}$

[4 marks]

Question 7

By using the binomial expansion, or otherwise, show that,

$$(1 + 2i)^5 = 41 - 38i$$

[4 marks]

Question 8

The complex number z is defined by,

$$z = \frac{3 + qi}{q - 5i} \quad \text{where } q \in \mathbb{R}$$

Given that the real part of z is $\frac{1}{13}$,

(i) find the possible values of q

[5 marks]

(ii) write the possible values of z in the form $a + bi$ where a and b are real constants.

[3 marks]

Question 9

$$z = 4 - i\sqrt{2}$$

Use algebra to express $\frac{z + 4}{z - 3}$ in the form $p + qi\sqrt{2}$
where p and q are rational numbers.

[5 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk