A-Level Pure Mathematics

Year 1, Year 2 Revision

Grade Grabber 2

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 26

Question 1

A geometric series has first term $\sin 2\theta$ and common ratio $\cos 2\theta$

where $0 < \theta < \frac{\pi}{4}$

Show that the sum to infinity of this series is $\cot \theta$

[4 marks]

Question 2

Consider the equation;

$$13\sin\theta + 8\cos\theta = 11 \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

This question is about solving this equation in two different ways, one approximate, the other exact.

(a) You are reminded that if θ is small and measured in radians,

- $\sin\theta \approx \theta$
- $\cos\theta \approx 1 - \frac{\theta^2}{2}$
- $\tan\theta \approx \theta$

Use the small angle approximations to show that;

$$4\theta^2 - 13\theta + 3 = 0$$

and hence find a valid approximate solutions to

$$13\sin\theta + 8\cos\theta = 11 \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

[4 marks]

(b) Assume that

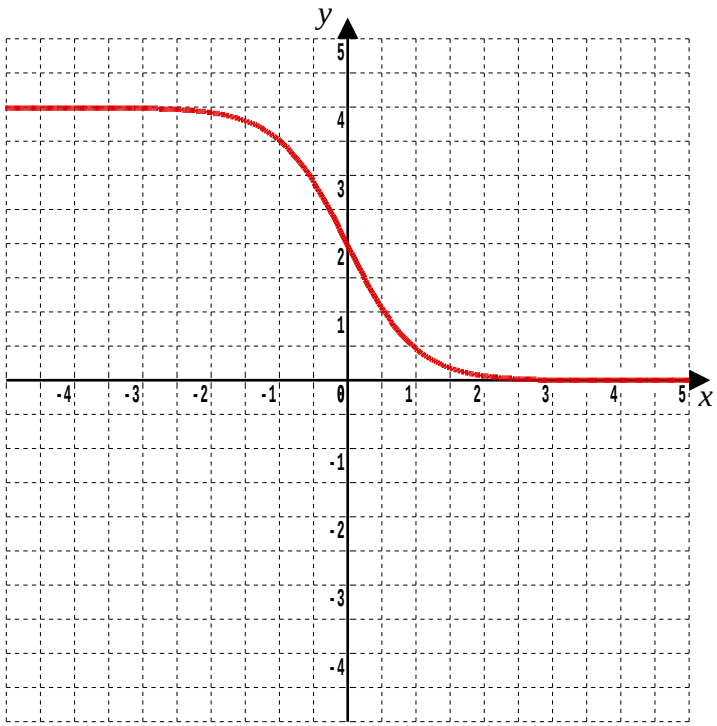
$$13 \sin\theta + 8\cos\theta = R\sin(\theta + \alpha) \qquad 0 \leqslant \alpha \leqslant \frac{\pi}{2}$$

Determine the value of α and the value of R and hence solve

$$13 \sin\theta + 8\cos\theta = 11 \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

[4 marks]

Question 3



The graph shows a plot of the curve with equation

$$y = \frac{4}{e^{2x} + 1} \qquad x \in R$$

The area of a finite region, bounded by the curve, the y-axis, the x-axis, and the line with equation $x = 0.5$ is of interest.

(i) On the diagram, shade in the area of interest.

[1 mark]

(ii) Use the substitution $u = e^{2x}$ to show that the area of interest can be given by

$$\int_a^b \frac{2}{u\,(u + 1)}\,du$$

where a and b are constants to be found

[3 marks]

(iii) Hence use algebraic integration to show that the exact area of the region of interest is

$$Area = ln\left(\frac{2e}{e+1}\right)^2$$

[5 marks]

Question 4

One root of the following equation is 2;

$$x^3 + kx + 4 = 0$$

(i) Find the value of k

[1 mark]

(ii) Find the exact values of the other two roots.

[4 marks]