Lesson 7

A-Level Pure Mathematics : Year 2 Differential Equations I

7.1 Revision

Any solution based entirely on graphical or numerical methods is not acceptable Marks Available : 40

Question 1

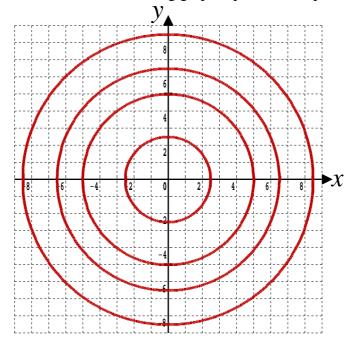
(i) Determine the general solution to the differential equation, $\frac{dy}{dx} = -\frac{x}{y}$

[3 marks]

(ii) The general solution is represented by all possible circles centred on the origin. Given that y = 6 when x = 2.5 find the particular solution. Present your solution in the form $x^2 + y^2 = r^2$ where *r* is a constant.

[1 mark]

(iii) Which circle on the following graph represents the particular solution ?



[1 mark]

A-Level Examination Question from June 2012, Paper C4, Q4 (Edexcel) Given that y = 2 at $x = \frac{\pi}{4}$, solve the differential equation,

$$\frac{dy}{dx} = \frac{3}{y \cos^2 x}$$

A-Level Examination Question from October 2020, Paper 1, Q14 (Edexcel) A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume $V \text{ cm}^3$

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that $\frac{dr}{dt} = -\frac{k}{r^2}$ where k is a positive constant.

[3 marks]

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty
- (**b**) solve the differential equation to find a complete equation linking r and t

[5 marks]

(c) Find the limitation on the values of *t* for which the equation in part (b) is valid.

[2 marks]

A-Level Examination Question from June 2011, Paper C4, Q8 (Edexcel)

(**a**) Find
$$\int (4y + 3)^{-\frac{1}{2}} dy$$

[2 marks]

(**b**) Given that y = 1.5 at x = -2, solve the differential equation,

$$\frac{dy}{dx} = \frac{\sqrt{(4y+3)}}{x^2}$$

giving your answer in the form y = f(x)

[6 marks]

A-Level Examination Question from January 2011, Paper C4, Q3 (Edexcel)(a) Express in partial fractions;

$$\frac{5}{(x-1)(3x+2)}$$

[3 marks]

(**b**) Hence find;
$$\int \frac{5}{(x-1)(3x+2)} dx, \qquad x > 1$$

[3 marks]

(c) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{dy}{dx} = 5y, \qquad x > 1$$

for which y = 8 at x = 2Give your answer in the form y = f(x)

[6 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk