Lesson 2

2.1 A Crucial Result

$$P(X \ge k) = 1 - P(X \le k - 1)$$

2.2 An Example Using The Crucial Result

Given that $X \sim B(25, 0.4)$, determine P ($X \ge 9$)

2.3 Example

The standard skin cream treatment for a particular medical skin condition has 0.8 probability of success.

Dr Spot has undertaken research in this area and has produced a new skin cream which has been successful with 28 out of 30 patients in a medical trial.

Dr Spot claims that the new cream is a 'medical advance'; an improvement on the standard skin cream treatment.

Test, at the 5% significance level, the claim made by Dr Spot.

2.4 Exercise

Question 1

A single observation, x, is taken from a binomial distribution B(40, p) where p is thought to be 0.25

- (i) What is the expected value of x?
- (ii) The value of x obtained is higher than expected, x = 15. Use this observation to test H₀ : p = 0.25 against H₁ : p > 0.25Use a 5% significance level.

Question 2

A single digit random number generator, generates integers between 0 and 9 It is suspected of being biased in favour of the number 0

It is about to generate 40 random digits.

- (i) How many 0s are expected, assuming the random number generator is not biased ?
- (**ii**) Set up a hypothesis test, by clearly stating the null hypothesis and the alternative hypothesis.

(iii)	Here are the 40 numbers generated;								
	6	1	0	8	4	1	0	0	7
	3	0	3	9	5	9	7	9	6
	8	8	0	7	3	7	2	6	0
	4	8	9	1	3	4	8	1	9
	2	0	6	0					

Is there statistical evidence at the 5% significance level, to support the suspicion ?

Question 3

A psychologist is doing research in the colour preferences of five year old girls. In one experiment he offers each of 20 girls a choice from three otherwise identical balls; one pink, one yellow and one blue.

Ten of the girls choose the pink.

He deduces that, of these three colours, five year old girls have a preference for pink. Is this conclusion justified ?

In your answer clearly state the distribution used along with the null and the alternative hypothesis employed and the significance level used.

Question 4

A dice is rolled 30 times.

- (i) How many rolls of a six are expected ?
- (ii) What is the fewest number of rolls of a six needed to persuade you, at the 5% significance level, that the dice is biased in favour of a six ?

Take care with 'the crucial result' in giving your final answer.

Question 5

A spinner with eight identical octagonal numbered sectors (as shown) is to be spun 48 times. It is suspected of being biased in favour of the number 3 sector.



- (i) What is the expected number of 3s, assuming the spinner is not biased ?
- (ii) What is the fewest number of 3s needed to persuade you, at the 5% significance level, that the spinner is biased in favour of a 3 ?
 Clearly state the null and alternative hypothesis used.

The *critical region* is the region of the probability distribution which, if the test statistic falls within it, would cause the null hypothesis to be rejected.

(iii) State the critical region for question 5

The *critical value* is the first value to fall inside of the critical region.

- (**iv**) State the critical value for question 5
- (v) When the spinner is spun, sure enough, more 3s than the expected number are obtained. In fact, 15 of the 48 spins resulted in a 3. Without further calculation, does this observation cause the null hypothesis to be rejected or not ?

Question 6

A random variable, *X*, has a distribution $X \sim B(30, 0.35)$. With $H_0: p = 0.35$ against $H_1: p > 0.35$ and using a 5% level of significance, find the critical region of this test.

Question 7

Given a binomial distribution $X \sim B(n, p)$, explain what is meant by;

- (i) A hypothesis test
- (**ii**) A null hypothesis
- (iii) A critical value
- (iv) An acceptance region
- (v) A 5% significance level in a one tailed test

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