### 2.1 A Crucial Result

$$
P(X \geqslant k)=1-P(X \leqslant k-1)
$$

### 2.2 An Example Using The Crucial Result

Given that $X \sim \mathrm{~B}(25,0.4)$, determine $\mathrm{P}(X \geqslant 9)$

### 2.3 Example

The standard skin cream treatment for a particular medical skin condition has 0.8 probability of success.
Dr Spot has undertaken research in this area and has produced a new skin cream which has been successful with 28 out of 30 patients in a medical trial.
Dr Spot claims that the new cream is a 'medical advance'; an improvement on the standard skin cream treatment.
Test, at the $5 \%$ significance level, the claim made by Dr Spot.

### 2.4 Exercise

## Question 1

A single observation, $x$, is taken from a binomial distribution $\mathrm{B}(40, p)$ where $p$ is thought to be 0.25
(i) What is the expected value of $x$ ?
( ii ) The value of $x$ obtained is higher than expected, $x=15$. Use this observation to test $\mathrm{H}_{0}: p=0.25$ against $\mathrm{H}_{1}: p>0.25$ Use a 5\% significance level.

## Question 2

A single digit random number generator, generates integers between 0 and 9
It is suspected of being biased in favour of the number 0
It is about to generate 40 random digits.
(i) How many 0 s are expected, assuming the random number generator is not biased?
( ii ) Set up a hypothesis test, by clearly stating the null hypothesis and the alternative hypothesis.
( iii ) Here are the 40 numbers generated;

| 6 | 1 | 0 | 8 | 4 | 1 | 0 | 0 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 0 | 3 | 9 | 5 | 9 | 7 | 9 | 6 |
| 8 | 8 | 0 | 7 | 3 | 7 | 2 | 6 | 0 |
| 4 | 8 | 9 | 1 | 3 | 4 | 8 | 1 | 9 |
| 2 | 0 | 6 | 0 |  |  |  |  |  |

Is there statistical evidence at the $5 \%$ significance level, to support the suspicion?

## Question 3

A psychologist is doing research in the colour preferences of five year old girls. In one experiment he offers each of 20 girls a choice from three otherwise identical balls; one pink, one yellow and one blue.
Ten of the girls choose the pink.
He deduces that, of these three colours, five year old girls have a preference for pink. Is this conclusion justified ?

In your answer clearly state the distribution used along with the null and the alternative hypothesis employed and the significance level used.

## Question 4

A dice is rolled 30 times.
( i ) How many rolls of a six are expected?
(ii) What is the fewest number of rolls of a six needed to persuade you, at the $5 \%$ significance level, that the dice is biased in favour of a six ?

Take care with 'the crucial result' in giving your final answer.

## Question 5

A spinner with eight identical octagonal numbered sectors (as shown) is to be spun 48 times. It is suspected of being biased in favour of the number 3 sector.

(i) What is the expected number of 3 s , assuming the spinner is not biased?
( ii ) What is the fewest number of 3 s needed to persuade you, at the 5\% significance level, that the spinner is biased in favour of a 3 ? Clearly state the null and alternative hypothesis used.

The critical region is the region of the probability distribution which, if the test statistic falls within it, would cause the null hypothesis to be rejected.
( iii ) State the critical region for question 5

The critical value is the first value to fall inside of the critical region.
( iv ) State the critical value for question 5
( v ) When the spinner is spun, sure enough, more 3 s than the expected number are obtained. In fact, 15 of the 48 spins resulted in a 3.
Without further calculation, does this observation cause the null hypothesis to be rejected or not?

## Question 6

A random variable, $X$, has a distribution $X \sim \mathrm{~B}(30,0.35)$.
With $\mathrm{H}_{0}: p=0.35$ against $\mathrm{H}_{1}: p>0.35$ and using a $5 \%$ level of significance, find the critical region of this test.

## Question 7

Given a binomial distribution $X \sim \mathrm{~B}(n, p)$, explain what is meant by;
(i) A hypothesis test
( ii ) A null hypothesis
(iii) A critical value
(iv) An acceptance region
( v ) A 5\% significance level in a one tailed test

