Lesson 8

Additional Mathematics
A-Level Pure Mathematics : Year 1
Binomial Expansion

8.1 The Binomial Theorem

Previously,[†] expanding the brackets of $(3 - 2x)^4$ was tackled.
Here is a reminder of the recommended method of setting out a solution;

$$\begin{aligned}(3-2x)^4 = {} & 1 \times (3)^4 \times (-2x)^0 \\ & + 4 \times (3)^3 \times (-2x)^1 \\ & + 6 \times (3)^2 \times (-2x)^2 \\ & + 4 \times (3)^1 \times (-2x)^3 \\ & + 1 \times (3)^0 \times (-2x)^4\end{aligned}$$

$$\therefore \quad (3-2x)^4 = 81 - 216x + 216x^2 - 96x^3 + 16x^4$$

This method can be generalised to give The Binomial Theorem;

$$\begin{aligned}(a+b)^n = {} & {}^nC_0 \times (a)^n \times (b)^0 \\ & + {}^nC_1 \times (a)^{n-1} \times (b)^1 \\ & + {}^nC_2 \times (a)^{n-2} \times (b)^2 \\ & + {}^nC_3 \times (a)^{n-3} \times (b)^3 \\ & + \ldots \\ & + {}^nC_r \times (a)^{n-r} \times (b)^r \\ & + \ldots \\ & + {}^nC_{n-1} \times (a)^1 \times (b)^{n-1} \\ & + {}^nC_n \times (a)^0 \times (b)^n\end{aligned}$$

The Binomial Theorem (for integer n)

$$(a+b)^n = a^n + {}^nC_1\, a^{n-1} b + \ldots + {}^nC_r\, a^{n-r} b^r + \ldots + b^n$$

This is provided to candidates in Additional Mathematics and A-Level exams

† In Lesson 4. Example 4.2, along with a Teaching Video solution

8.2 Exercise

Marks Available : 40

Question 1

Expand the brackets

$$\left(5x - x^2\right)^3 = \quad 1 \times (\quad) \times (\quad)$$

$$+ \quad 3 \times (\quad) \times (\quad)$$

$$+ \quad 3 \times (\quad) \times (\quad)$$

$$+ \quad 1 \times (\quad) \times (\quad)$$

So,

$$\left(5x - x^2\right)^3 =$$

[5 marks]

Question 2

In mathematics the pling symbol "!" means "factorial"

$5! = 5 \times 4 \times 3 \times 2 \times 1$

Check, on your calculator, that 5! = 120 (Using the special button marked "!")

Work out

(i) 6! **(ii)** 10! **(iii)** 13!

[3 marks]

Question 3

My calculator gives an error message when I try to work out 94!

The answer is way too big for my calculator to handle.

What is the smallest number, x, for which your calculator cannot calculate x! ?

[1 mark]

Question 4

Given calculator limitations, there is no point using one to work out

$$\frac{5000!}{4999!}$$

However, the answer, if you think about it, is easy to obtain using brain power.

What is the answer ?

[2 marks]

Question 5

Work out the following using a mixture of cunning and calculator

(i) $\frac{100!}{99\,!}$ **(ii)** $\frac{101!}{98!}$ **(iii)** $\frac{2021!}{2018!}$

[1, 2, 3 marks]

Question 6

Simplify

$$\frac{(n+4)!}{(n+1)!}$$

[3 marks]

Question 7

Simplify

$$\frac{(n+1)!}{(n-1)!}$$

[3 marks]

Question 8
The numbers in Pascal's Triangle are given by

$$^{n}C_{r} = \frac{n!}{r!\ (n-r)!}$$

Use this to derive a simplified expression for $^{n}C_{2}$

[4 marks]

Question 9
Expand the brackets;

$$(2 + x)(4 + 5x)^{3}$$

[8 marks]

Question 10

Further Mathematics Specimen Exam Paper 1, June 2020, Q16 (AQA)

The coefficient of the x^4 term in the expansion of $(2x + a)^6$ is 60

Work out the possible values of a

[5 marks]