**Lesson 7**

**Additional Mathematics**
**A-Level Pure Mathematics : Year 1**
**Binomial Expansion**

## 7.1 Investigating Binomial Probability

Binomial means “two state”: A real life “two state” situation is the flipping of a coin.

Suppose a biased coin is weighted with a probability of 0.36 of landing tails.
It is flipped 8 times. What is the probability that it lands tails exactly twice ?

To answer this question we begin by wanting to know how many different ways 2 tails and 6 heads can occur.

H H H H H H T T

H H H H H T H T
H H H H H T T H

**Method 1** : List them and observe the list has 28 entries
Takes time and is tricky to get correct.

H H H H T H H T
H H H H T H T H
H H H H T T H H

**Method 2** : Write out Pascal's Triangle
Observe that Row 8, Column 2 is 28
Less time but a tedious job for just one number

H H H T H H H T
H H H T H H T H
H H H T H T H H
H H H T T H H H

H H T H H H H T
H H T H H H T H
H H T H H T H H
H H T H T H H H
H H T T H H H H

H T H H H H H T
H T H H H H T H
H T H H H T H H
H T H H T H H H
H T H T H H H H
H T T H H H H H

T H H H H H H T
T H H H H H T H
T H H H H T H H
T H H H T H H H
T H H T H H H H
T H T H H H H H
T T H H H H H H

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | 1 | | | | |
| | | | 1 | | 1 | | | |
| | | 1 | | 2 | | 1 | | |
| | 1 | | 3 | | 3 | | 1 | |
| 1 | | 4 | | 6 | | 4 | | 1 |
| | 1 | | 5 | | 10 | | 5 | |
| 1 | | 6 | | 15 | | 20 | | 15 |
| | 1 | | 7 | | 21 | | 35 | |
| 1 | | 8 | | 28 | | 56 | | 70 |

**Method 3** : Use a calculator

$$^{8}C_{2} = 28$$

**Having got the 28...**
Each of the 28 items in the list will be a multiplication of $0.36 \times 0.36 \times 0.64 \times 0.64 \times 0.64 \times 0.64 \times 0.64 \times 0.64$ (with those same numbers occurring in different orders)
Thus the quick calculation is;

$$28 \times 0.36^{2} \times 0.64^{6}$$
$$= 28 \times 0.1296 \times 0.0687$$
$$= 0.249 \text{ (Give answers to 3 decimal places)}$$

So, when this biased coin is flipped eight times, there is a 25% probability it will land tails exactly twice.

**7.2 Exercise**

Marks Available : 60

**Question 1**

Here is Pascal's Triangle, left justified.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

**( i )** Circle the entry in Row 7, Column 5
*Notice this is in the 8th row and the 6$^{th}$ column !*

**[ 1 mark ]**

**( ii )** Use your calculator to determine ${}^{7}C_{5}$
*This should be your part **( i )** answer*

**[ 1 mark ]**

**Question 2**

Here is Pascal's Triangle, left justified.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

**( i )** Circle the entry in Row 4, Column 0

**[ 1 mark ]**

**( ii )** Circle the entry in Row 5, Column 5

**[ 1 mark ]**

**( iii )** Circle the entry in Row 9, Column 3

**[ 1 mark ]**

**( iv )** Find a solution pair ( $n$, $r$ ) to

$${}^{n}C_{r} = 70$$

**[ 1 mark ]**

**Question 3**

Use your calculator to determine ${}^{13}C_5$

**[ 1 mark ]**

**Question 4**

A coin is flipped 6 times.

**( i )** Describe which calculator buttons you would press in order to determine the number of ways exactly 2 tails could be obtained ?

**[ 1 mark ]**

**( ii )** Write out enough of Pascal's Triangle so that you can then draw a circle around the number in it, corresponding to your part (i) answer

**[ 2 marks ]**

**Question 5**

A coin is flipped 20 times.
In how many ways can exactly 13 tails be obtained ?
HINT : Use a calculator.

**[ 2 marks ]**

**Question 6**

A coin is flipped 40 times.
In how many ways can exactly 5 heads be obtained ?

**[ 2 marks ]**

**Question 7**

A biased coin is weighted such that it has a probability of 0.45 of landing tails.
It is flipped 8 times. What is the probability that it lands tails exactly thrice ?
Give your answer to 3 decimal places.

**[ 4 marks ]**

**Question 8**

A typist has a probability of 0.99 of typing each letter in a sentence correctly.
What is the percentage probability of exactly two mistakes in a sentence containing 180 letters, if mistakes are made at random ? Give your answer to 3 decimal places.

**[ 4 marks ]**

**Question 9**

In a box of Smarties™ there are eight different colours which normally occur in equal proportions. Sebastian is given 24 Smarties™, and blue ones are his favourite. Assume these come from a very large box.

**( i )** How many blue Smarties™ would he expect to get (on average) ?

**[ 1 mark ]**

**( ii )** What is the probability that he gets this number ?

**[ 4 marks ]**

**( iii )** What is the probability that he gets fewer than expected ?

**[ 4 marks ]**

**( iv )** What is the probability that he gets more than expected ?

**[ 1 mark ]**

**( v )** Explain why the assumption was made that the 24 Smarties™ given to Sebastian came from a very large box ?

**[ 2 marks ]**

**Question 10**

**( i )** Determine ${}^{6}C_{2}$

**[ 1 mark ]**

**( ii )** Determine ${}^{6}C_{4}$

**[ 1 mark ]**

**( iii )** Explain with the help of Pascal's Triangle why ${}^{6}C_{2} = {}^{6}C_{4}$

**[ 2 marks ]**

**( iv )** Explain, in your own words, why ${}^{n}C_{0} = {}^{n}C_{n}$

**[ 2 marks ]**

**( v )** Explain, in your own words, why ${}^{n}C_{2} = {}^{n}C_{n-2}$

**[ 2 marks ]**

**( vi )** What formula can be written down for ${}^{n}C_{m}$ that generalises the observations made above ?

**[ 2 marks ]**

*The next question is about obtaining the probability distribution curve for a simple coin flipping situation.*

**Question 11**

A biased coin is weighted such that it has a probability of 0.4 of landing tails.
It is flipped 6 times.

**( i )** Show that the probability of exactly 4 tails being obtained is 0.138.
i.e. About 14% probable.

**[ 2 marks ]**

**( ii )** Work out the probability of exactly 0, 1, 2, 3, 5 and 6 tails being obtained.
Present your solutions in the table below.

| N° of tails | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Probability | | | | | 14% | | |

**[ 8 marks ]**

**( iii )** Present your table of results as an accurate bar chart.

**[ 6 marks ]**