Further Mechanics A-Level Mathematics
Optional Course Component
Further Mechanics 1

KINEMATICS

V

An artists impression of NASAs Orion spacecraft, on it's Artemis I mission, November 2022

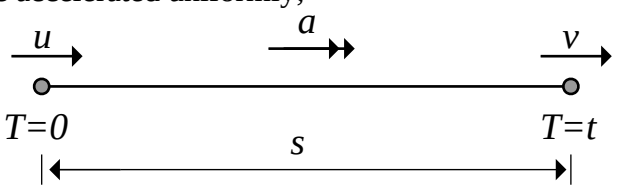
K I N E M A T I C S
V

Lesson 1

Further A-Level Mathematics, Mechanics 1
Kinematics V

1.1 Revisiting the five SUVAT equations

Our study of kinematics began when we considered a straight line interval over which a particle accelerated uniformly;



s = *displacement*

u = *initial velocity*

v = *final velocity*

a = *acceleration (constant)*

t = *time*

$v = u + at$	s
$s = vt - \frac{1}{2} a t^2$	u
$s = ut + \frac{1}{2} a t^2$	v
$s = \left(\frac{v + u}{2} \right) t$	a
$v^2 = u^2 + 2as$	t

1.2 Revisiting Variable Acceleration

We then took this a step further and looked at situations in which the acceleration was NOT uniform, in which case the SUVAT equations did not apply.
In such situations we used the calculus of differentiation and integration to move between the quantities of displacement, s, velocity, v, and acceleration, a.

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$

$$s = \int v \, dt \qquad v = \int a \, dt$$

When integrating it is crucial to remember the constant of integration, often not zero.

1.3 Variable Acceleration Equations As Functions Of Time

Our grasp of the pure mathematics integration techniques means that we can now tackle more sophisticated versions of these problems in which the variable acceleration is a function of time. In a future lesson we'll look at problems in which the variable acceleration is a function, not of time, but of displacement.

1.4 Example

A particle P is moving in a straight line.
Initially P is moving through a point O with speed 2 $m.s^{-1}$
At time t seconds after passing through O the acceleration of P is $6e^{-2t}$ $m.s^{-2}$ in the direction OP.

(i) Find an expression for the velocity of the particle at time t seconds

(ii) Find the velocity when $t = 1$ s, $t = 2$ s and $t = 3$ s

(iii) Sketch the velocity-time graph for particle *P's* motion

(iv) Comment on how this velocity-time graph is more realistic than those of the GCSE course.

1.5 Exercise

Question 1

A particle P is moving in a straight line.
Initially P is moving through a point O with speed 4 $m.s^{-1}$
At time t seconds after passing through O the acceleration of P is $3e^{-0.25t}$ $m.s^{-2}$ in the direction OP.

(i) Find an expression for the velocity of the particle at time t seconds

(ii) Find, accurate to three significant figures, the velocity when $t = 8$ s

(iii) Sketch the velocity-time graph for particle *P's* motion

Question 2

A particle P is moving along the x-axis in the direction of x-increasing.
At time t seconds, the velocity of P is $t\,e^{t}$ m.s^{-1}

When $t = 0$, P is at the origin.

(i) Find an expression for the location of P at time, t

(ii) When $t = 4$, show that P is more than 160 metres from O

Question 3

A particle P is moving along the x-axis in the direction of x-increasing.
At time t seconds, the velocity of P is $t \sin t$ m.s^{-1}
When $t = 0$, P is at the origin.

(i) Find an expression for the location of P at time, t

(ii) When $t = \frac{\pi}{2}$, show that P is exactly 1 metre from O

(iii) Previously, when $t = \frac{\pi}{3}$, was P less far from O than when $t = \frac{\pi}{2}$?

Question 4

At time t seconds the velocity, v m.s^{-1}, of a particle moving in a straight line is given by;

$$v = \frac{4}{3 + 2t} \qquad t \geqslant 0$$

When $t = 0$, the particle is at a point A

When $t = 3$, the particle is at the point B

Show that the exact distance between A and B is $2\ ln3$

Question 5

A particle P is moving along a straight line.
Initially P is at rest.
At time t seconds P has velocity v m.s^{-1} and acceleration a ms^{-2} where

$$a = \frac{6t}{(2 + t^2)^2} \qquad t \geqslant 0$$

(i) Find an expression for v in terms of t

(ii) Find v when $t = 1$

(iii) Find v when $t = 2$

(iv) Sketch the velocity-time graph for P

Question 6

A particle P is moving along a straight line

When $t = 0$, P is passing through a point A

At time t seconds after passing through A the velocity v m.s^{-1} of P is given by

$$v = e^{2t} - 11e^{t} + 15t$$

Find

(i) the values of t for which the acceleration is zero

(ii) the distance of P from A when $t = \ln 3$