Lesson 6

A-Level Pure Mathematics : Year 2 Integration II

6.1 Beyond The Power Function

Suitable integrals, *I*, that can be determined by applying The Chain Rule Backwards have been spotted by identifying the key situation;

$$I = a \int f'(x) [f(x)]^n dx$$
 where *n* and *a* are constants

That power of n can be thought of as a function in its own right.

With $g(x) = x^n$ another way of expressing what we've been looking for is;

$$I = a \int f'(x) g[f(n)]$$

There is no reason why function g has to be a power function; The Chain Rule Backwards can be wielded when g is other functions such as cos or sin

The Chain Rule Backwards

$$\int f'(x) [f(x)]^n dx = \frac{[f(x)]^{n+1}}{(n+1)} + c \qquad n \neq -1$$

$$\int f'(x) [f(x)]^{-1} dx = ln | f(x)| + c \qquad \text{i.e. with } n = -1$$

$$\int f'(x) \cos[f(x)] dx = \sin[f(x)] + c$$

$$\int f'(x) \sin[f(x)] dx = -\cos[f(x)] + c$$

$$\int f'(x) e^{[f(x)]} dx = e^{[f(x)]} + c$$

Example

Determine: $\int 63 x^2 e^{7x^3} dx$

Teaching Video: http://www.NumberWonder.co.uk/v9045/6.mp4



[3 marks]

6.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable Marks Available : 40

Question 1

Determine $\int 30x \, e^{5x^2} \, dx$

[3 marks]

Question 2

Evaluate: $\int_0^2 x^2 e^{x^3} dx$

Give the answer to the nearest integer.

[4 marks]

Determine: $\int 100 x^3 \cos(6 + 5 x^4) dx$

[4 marks]

Question 4

Determine: $\int \frac{3}{2} x \sin(x^2) dx$

[4 marks]

Question 5

Determine :

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$$

[4 marks]

The graph is of the function $f(x) = \frac{12}{\sqrt{3x+1}}$



Find the area shown shaded in the graph which is bound by the curve, the *x*-axis and the vertical lines x = 1 and x = 5



Find the exact area of the 'bump' shown shaded in the graph which is bounded by g(x), the *x*-axis and the lines x = 0 and $x = 2\sqrt{\pi}$

$$f(x) = e^{\sqrt{x}}$$

(i) Find f'(x)

[2 marks]

(**ii**) Evaluate to 3 decimal places;

$$\int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

[5 marks]

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