## Lesson 6

## A-Level Pure Mathematics : Year 2

Integration II

### 6.1 Beyond The Power Function

Suitable integrals, $I$, that can be determined by applying The Chain Rule Backwards have been spotted by identifying the key situation;

$$
I=a \int f^{\prime}(x)[f(x)]^{n} d x \quad \text { where } n \text { and } a \text { are constants }
$$

That power of $n$ can be thought of as a function in its own right.
With $g(x)=x^{n}$ another way of expressing what we've been looking for is;

$$
I=a \int f^{\prime}(x) g[f(n)]
$$

There is no reason why function $g$ has to be a power function; The Chain Rule Backwards can be wielded when $g$ is other functions such as $\cos$ or $\sin$

## The Chain Rule Backwards

$$
\begin{array}{rlrl}
\int f^{\prime}(x)[f(x)]^{n} d x & =\frac{[f(x)]^{n+1}}{(n+1)}+c & n \neq-1 \\
\int f^{\prime}(x)[f(x)]^{-1} d x & =\ln |f(x)|+c & \text { i.e. with } n=-1 \\
\int f^{\prime}(x) \cos [f(x)] d x & =\sin [f(x)]+c & \\
\int f^{\prime}(x) \sin [f(x)] d x & =-\cos [f(x)]+c & \\
\int f^{\prime}(x) e^{[f(x)]} d x & =e^{[f(x)]}+c &
\end{array}
$$

## Example

Determine: $\int 63 x^{2} e^{7 x^{3}} d x$

Teaching Video: http://www.NumberWonder.co.uk/v9045/6.mp4


### 6.2 Exercise

> Any solution based entirely on graphical or numerical methods is not acceptable Marks Available : 40

## Question 1

Determine $\int 30 x e^{5 x^{2}} d x$

## Question 2

Evaluate: $\int_{0}^{2} x^{2} e^{x^{3}} d x$
Give the answer to the nearest integer.

## Question 3

Determine: $\int 100 x^{3} \cos \left(6+5 x^{4}\right) d x$

## [ 4 marks ]

Question 4
Determine: $\int \frac{3}{2} x \sin \left(x^{2}\right) d x$

## Question 5

Determine : $\quad \int \frac{\cos (\sqrt{x})}{\sqrt{x}} d x$

## Question 6

The graph is of the function $f(x)=\frac{12}{\sqrt{3 x+1}}$


Find the area shown shaded in the graph which is bound by the curve, the $x$-axis and the vertical lines $x=1$ and $x=5$

## Question 7

The graph is of the function, $g(x)=x \sin \left(\frac{x^{2}}{4}\right)$


Find the exact area of the 'bump' shown shaded in the graph which is bounded by $g(x)$, the $x$-axis and the lines $x=0$ and $x=2 \sqrt{\pi}$

## Question 8

$$
f(x)=e^{\sqrt{x}}
$$

(i) Find $f^{\prime}(x)$
( ii ) Evaluate to 3 decimal places;

$$
\int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} d x
$$

