Lesson 3

A-Level Pure Mathematics : Year 2 Integration II

3.1 Unlocking The Backwards



The Chain Rule Backwards

$$\int f'(x) [f(x)]^n dx = \frac{[f(x)]^{n+1}}{(n+1)} + c \qquad n \neq -1$$

Initial problems involving The Chain Rule Backwards kept f(x) as a simple linear function. That is, f(x) = mx + cHowever, this limitation will now be removed !

Example N° 1

Determine: $\int 120 x^3 (3x^4 + 7)^4 dx$

Teaching Video: http://www.NumberWonder.co.uk/v9045/3a.mp4



[3 marks]

Example N° 1 is all that's needed for the first six questions in Exercise 3.2 So you may prefer to do those questions first then watch the Example N° 2 video ahead of Question 7 onwards, for what Example N° 2 will help.

Example N° 2

Determine: $\int 64(x^3 - 2)(x^4 - 8x)^3 dx$

Teaching Video: http://www.NumberWonder.co.uk/v9045/3b.mp4



[4 marks]

3.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable Marks Available : 40

In each question use The Chain Rule Backwards to perform the integration given. Most questions will require that a "fiddle factor" be introduced.

Question 1

Determine: $\int 150x (3x^2 + 1)^4 dx$

Determine: $\int 56x (x^2 - 5)^3 dx$

[3 marks]

Question 3

Determine: $\int 42 x^2 (2 - x^3)^6 dx$

[3 marks]

Question 4

Determine: $\int 12 x \sqrt{x^2 + 6} dx$

[3 marks]

Determine:
$$\int \frac{21 x}{(1 + x^2)^2} dx$$

[3 marks]

Question 6
Determine:
$$\int \frac{12 x^3}{\sqrt{3 + x^4}} dx$$

[3 marks]

Question 7

Determine: $\int 60(x^2 - 1)(x^3 - 3x + 7)^4 dx$

[4 marks]

Determine:
$$\int \frac{x+1}{(x^2+2x+3)^4} dx$$

[4 marks]

Question 9

Determine:
$$\int \frac{x^2 + 2}{\sqrt{x^3 + 6x}} dx$$

[4 marks]

(i) Differentiate $y = sin(x^3 + 9x)$

[3 marks]

(ii) With part (i) in mind, find
$$\int (x^2 + 3) \cos(x^3 + 9x) dx$$

[3 marks]

(iii) Determine:
$$\int x^2 \cos(4x^3 - 7) dx$$

[4 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School It may be freely duplicated and distributed, unaltered, for non-profit educational use In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**" © 2021 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk