### 3.1 Unlocking The Backwards



The Chain Rule Backwards

$$
\int f^{\prime}(x)[f(x)]^{n} d x=\frac{[f(x)]^{n+1}}{(n+1)}+c \quad n \neq-1
$$

Initial problems involving The Chain Rule Backwards kept $f(x)$ as a simple linear function. That is, $f(x)=m x+c$
However, this limitation will now be removed !

## Example $\mathbf{N}^{\circ} 1$

Determine: $\int 120 x^{3}\left(3 x^{4}+7\right)^{4} d x$

Teaching Video: http://www.NumberWonder.co.uk/v9045/3a.mp4


Example $\mathrm{N}^{\circ} 1$ is all that's needed for the first six questions in Exercise 3.2
So you may prefer to do those questions first then watch the Example $\mathrm{N}^{\circ} 2$ video ahead of Question 7 onwards, for what Example $\mathrm{N}^{\circ} 2$ will help.

## Example $\mathbf{N}^{\circ} 2$

Determine: $\int 64\left(x^{3}-2\right)\left(x^{4}-8 x\right)^{3} d x$

Teaching Video: http://www.NumberWonder.co.uk/v9045/3b.mp4

[ 4 marks ]

### 3.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable Marks Available : 40

In each question use The Chain Rule Backwards to perform the integration given. Most questions will require that a "fiddle factor" be introduced.

## Question 1

Determine: $\int 150 x\left(3 x^{2}+1\right)^{4} d x$

## Question 2

Determine: $\int 56 x\left(x^{2}-5\right)^{3} d x$

## Question 3

Determine: $\int 42 x^{2}\left(2-x^{3}\right)^{6} d x$

## Question 4

Determine: $\int 12 x \sqrt{x^{2}+6} d x$

## Question 5

Determine: $\int \frac{21 x}{\left(1+x^{2}\right)^{2}} d x$

## Question 6

Determine: $\int \frac{12 x^{3}}{\sqrt{3+x^{4}}} d x$

## Question 7

Determine: $\int 60\left(x^{2}-1\right)\left(x^{3}-3 x+7\right)^{4} d x$

## Question 8

Determine: $\int \frac{x+1}{\left(x^{2}+2 x+3\right)^{4}} d x$

## Question 9

Determine: $\int \frac{x^{2}+2}{\sqrt{x^{3}+6 x}} d x$

## Question 10

(i) Differentiate $y=\sin \left(x^{3}+9 x\right)$
(ii) With part (i) in mind, find $\int\left(x^{2}+3\right) \cos \left(x^{3}+9 x\right) d x$
(iii) Determine: $\int x^{2} \cos \left(4 x^{3}-7\right) d x$

