### Lesson 2

## A-Level Pure Mathematics : Year 2 Integration II

#### 2.1 Thinking Backwards

When  $y = \frac{(4+5x)^6}{6}$  is differentiated the result is  $\frac{dy}{dx} = \frac{6(4+5x)^5}{6} \times 5$  by the chain rule  $= 5(4+5x)^5$ 

When  $y = \frac{[f(x)]^{n+1}}{(n+1)}$  is differentiated the result is  $\frac{dy}{dx} = \frac{(n+1)[f(x)]^n}{(n+1)} \times f'(x)$  by the chain rule  $= f'(x) [f(x)]^n$ 

By The Fundamental Theorem of Calculus,

### The Chain Rule Backwards

$$\int f'(x) \left[ f(x) \right]^n dx = \frac{\left[ f(x) \right]^{n+1}}{(n+1)} + c$$

The two key consequences of this result are;

• Every time a new integration question is tackled, a mental scan must be made to se if it is in the form of a function raised to a power with the derivative of that function sitting in front.

In other words, watch out for  $f'(x) [f(x)]^n$ 

An alertness needs to be maintained for situations in which the desired set up of f'(x) [f(x)]<sup>n</sup> can be created by making use of a "fiddle factor". See example N° 2 and example N° 3.

## Example N° 1

Integrate the following:  $\int 7 (7x + 2)^3 dx$ 

[ 3 marks ]

**Solution**: It is spotted that with  $f(x) = 7x + 2 \implies f'(x) = 7$  and that this

situation is that of a chain rule backwards with n = 3

Thus, 
$$\int 7 (7x + 2)^3 dx = \frac{(7x + 2)^4}{4} + c$$

The teaching video will talk through Example  $N^\circ$  2 and Example  $N^\circ$  3

Teaching Video: <u>http://www.NumberWonder.co.uk/v9045/2.mp4</u>



Example N° 2

Integrate the following:  $\int (3x + 1)^4 dx$ 

[ 3 marks ]

# Example N° 3

Integrate the following:  $\int \frac{24}{(4x-2)^4} dx$ 

# 2.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable Marks Available : 50

Question 1

$$\int (3x+2)^5 dx$$

[ 3 marks ]

**Question 2** 

$$\int (7x-3)^3 dx$$

[ 3 marks ]

**Question 3** 

$$60\int \left(4+5x\right)^3 dx$$

[ 3 marks ]

**Question 4** 

$$\int \left(3 - 2x\right)^5 dx$$

$$\int \left(7 - \frac{1}{2}x\right)^6 dx$$

[ 3 marks ]

**Question 6** 

$$18 \int \sqrt{1 + 4x} \, dx$$

[ 3 marks ]

Question 7

$$\int \left(\frac{2x}{3} + 8\right)^{\frac{1}{2}} dx$$

$$\int \frac{1}{(2x-1)^4} \, dx$$

[ 3 marks ]

Question 9

$$\int 6 \left( 2x - 1 \right)^{-2} dx$$

[ 3 marks ]

Question 10

$$\int \frac{6}{\sqrt{3x - 1}} \, dx$$

$$\int \frac{1}{4(x+3)^2} dx$$

[ 3 marks ]

**Question 12** 

$$\int 14 \, (3+2x)^{-2} \, dx$$

[ 3 marks ]

**Question 13** 

$$\int \frac{1}{\left(1-2x\right)^{\frac{3}{2}}} dx$$

$$\int_{0}^{1} (4x + 1)^{4} dx$$

[ 3 marks ]

**Question 15** 

$$\int_0^4 \sqrt{2x+1} \ dx$$

[4 marks]

**Question 16** 

$$\int_{2}^{5} \frac{1}{(x-1)^{3}} \, dx$$

[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk