## A-Level Pure Mathematics : Year 2

Integration II

### 2.1 Thinking Backwards

When $y=\frac{(4+5 x)^{6}}{6}$ is differentiated the result is

$$
\begin{aligned}
\frac{d y}{d x} & =\frac{6(4+5 x)^{5}}{6} \times 5 \quad \text { by the chain rule } \\
& =5(4+5 x)^{5}
\end{aligned}
$$

When $y=\frac{[f(x)]^{n+1}}{(n+1)}$ is differentiated the result is

$$
\begin{aligned}
\frac{d y}{d x} & =\frac{(n+1)[f(x)]^{n}}{(n+1)} \times f^{\prime}(x) \text { by the chain rule } \\
& =f^{\prime}(x)[f(x)]^{n}
\end{aligned}
$$

By The Fundamental Theorem of Calculus,

## The Chain Rule Backwards

$$
\int f^{\prime}(x)[f(x)]^{n} d x=\frac{[f(x)]^{n+1}}{(n+1)}+c
$$

The two key consequences of this result are;

- Every time a new integration question is tackled, a mental scan must be made to se if it is in the form of a function raised to a power with the derivative of that function sitting in front.
In other words, watch out for $f^{\prime}(x)[f(x)]^{n}$
- An alertness needs to be maintained for situations in which the desired set up of $f^{\prime}(x)[f(x)]^{n}$ can be created by making use of a "fiddle factor". See example $\mathrm{N}^{\circ} 2$ and example $\mathrm{N}^{\circ} 3$.


## Example $\mathbf{N}^{\circ} 1$

Integrate the following: $\int 7(7 x+2)^{3} d x$

Solution : It is spotted that with $f(x)=7 x+2 \Rightarrow f^{\prime}(x)=7$ and that this situation is that of a chain rule backwards with $n=3$
Thus, $\int 7(7 x+2)^{3} d x=\frac{(7 x+2)^{4}}{4}+c$

The teaching video will talk through Example $\mathrm{N}^{\circ} 2$ and Example $\mathrm{N}^{\circ} 3$

Teaching Video: http://www.NumberWonder.co.uk/v9045/2.mp4


## Example $\mathbf{N}^{\circ} 2$

Integrate the following: $\quad \int(3 x+1)^{4} d x$

## Example $\mathbf{N}^{\circ} \mathbf{3}$

Integrate the following: $\quad \int \frac{24}{(4 x-2)^{4}} d x$

### 2.2 Exercise

> Any solution based entirely on graphical or numerical methods is not acceptable Marks Available : 50

## Question 1

$$
\int(3 x+2)^{5} d x
$$

## Question 2

$$
\int(7 x-3)^{3} d x
$$

Question 3

$$
60 \int(4+5 x)^{3} d x
$$

## Question 4

$$
\int(3-2 x)^{5} d x
$$

Question 5

$$
\int\left(7-\frac{1}{2} x\right)^{6} d x
$$

## Question 6

$$
18 \int \sqrt{1+4 x} d x
$$

Question 7

$$
\int\left(\frac{2 x}{3}+8\right)^{\frac{1}{2}} d x
$$

Question 8

$$
\int \frac{1}{(2 x-1)^{4}} d x
$$

## Question 9

$$
\int 6(2 x-1)^{-2} d x
$$

Question 10

$$
\int \frac{6}{\sqrt{3 x-1}} d x
$$

## Question 11

$$
\int \frac{1}{4(x+3)^{2}} d x
$$

Question 12

$$
\int 14(3+2 x)^{-2} d x
$$

Question 13

$$
\int \frac{1}{(1-2 x)^{\frac{3}{2}}} d x
$$

## Question 14

$$
\int_{0}^{1}(4 x+1)^{4} d x
$$

## Question 15

$$
\int_{0}^{4} \sqrt{2 x+1} d x
$$

## Question 16

$$
\int_{2}^{5} \frac{1}{(x-1)^{3}} d x
$$

