

**6.1 Vector Ratios**

When a point  $M$  is described as being the midpoint of the line  $AB$  the length of the line is divided in a ratio of,

$$AM : MB$$

$$\frac{1}{2} : \frac{1}{2}$$

or, as integers are often preferred, by multiplying through by 2,

$$AM : MB$$

$$1 : 1$$

Of course, there is no reason why other ratios could not be used such as, for example, a point  $X$  that divides the line  $AB$  in the ratio,

$$AX : XB$$

$$3 : 2$$

which could also be written, upon dividing through by 5,

$$AX : XB$$

$$\frac{3}{5} : \frac{2}{5}$$

This tells the reader that  $X$  is  $\frac{3}{5}$  of the way along line  $AB$  from  $A$

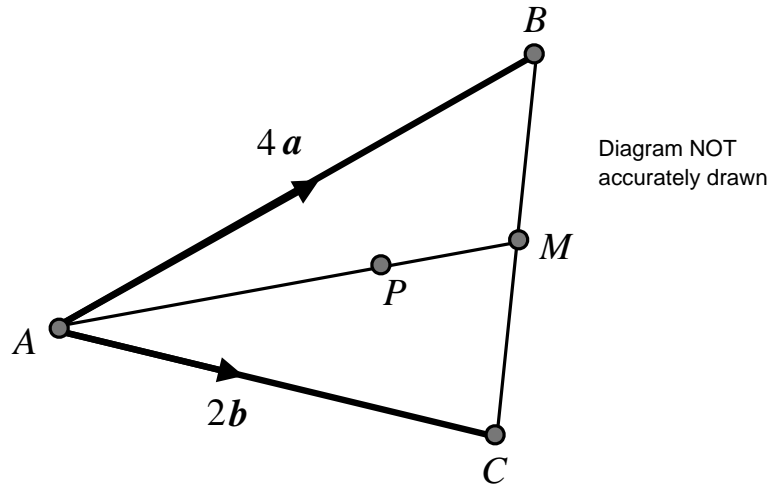
$$\begin{aligned} \text{If, for example, } \vec{AB} = 15s + 10t \text{ then } \vec{AX} &= \frac{3}{5} \vec{AB} \\ &= \frac{3}{5} (15s + 10t) \\ &= 9s + 6t \end{aligned}$$

□□□□□□□□□□□□□□□□△△△△△△△△△△	$15s + 10t$
□□□△△□□□△△□□□△△□□□△△□□□△△	$5(3s + 2t)$
□□□△△□□□△△□□□△△ + □□□△△□□□△△	$3(3s + 2t) + 2(3s + 2t)$
□□□△△□□□△△□□□△△ : □□□△△□□□△△	$3(3s + 2t) : 2(3s + 2t)$
3 : 2	3 : 2
□□□□□□□□□□△△△△△△△△ : □□□□□□△△△△△	$9s + 6t : 6s + 4t$

An interesting similarity between ratios of squares and triangles on the one hand and ratios of vectors  $s$  and  $t$  in the other

## 6.2 A Recent Examination Question

GCSE Examination Question from January 2020, Paper 2H, Q23



$ABC$  is a triangle in which the midpoint of  $BC$  is  $M$  and  $P$  is a point on  $AM$ .

$$\vec{AB} = 4\mathbf{a} \quad \vec{AC} = 2\mathbf{b} \quad \vec{AP} = \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$$

Find the ratio  $AP : PM$

Teaching Video : <http://www.NumberWonder.co.uk/v9009/6.mp4>



Watch the teaching video then write out a solution to the question.



[ 3 marks ]

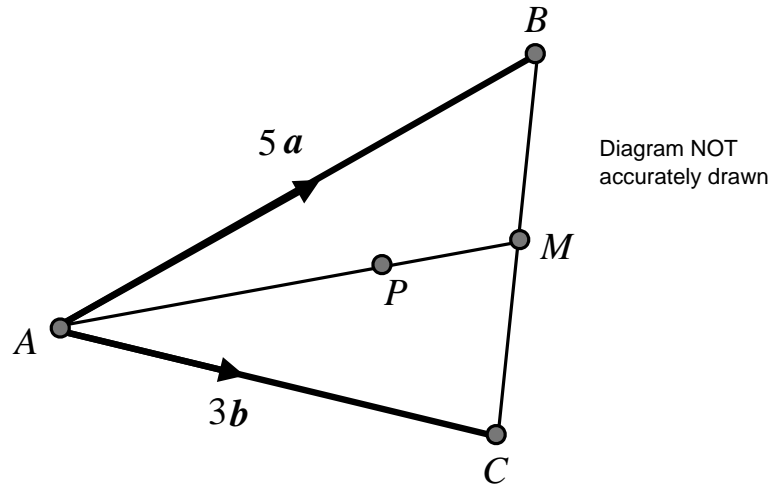
### 6.3 Exercise

*Any solution based entirely on graphical or numerical methods is not acceptable.*

*Make the method used clear.*

Marks available : 30

#### Question 1



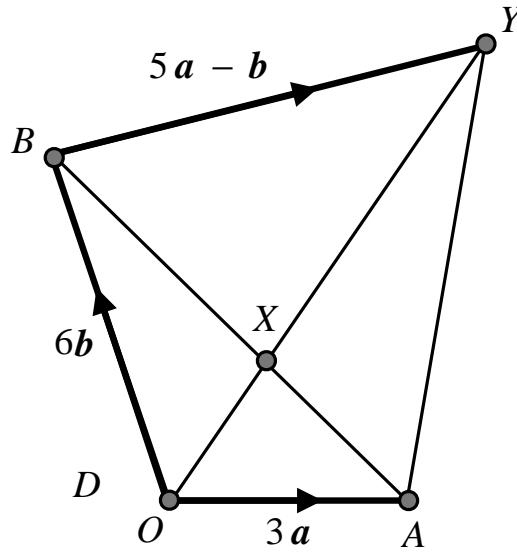
$ABC$  is a triangle in which the midpoint of  $BC$  is  $M$  and  $P$  is a point on  $AM$ .

$$\vec{AB} = 5a \quad \vec{AC} = 3b \quad \vec{AP} = \frac{5}{6}a + \frac{1}{2}b$$

Find the ratio  $AP : PM$

[ 3 marks ]

**Question 2**



$OAYB$  is a quadrilateral, with the diagonals  $AB$  and  $OY$  intersecting at point  $X$ .

The ratio  $AX : XB = 1 : 2$

$$\vec{OA} = 3a \quad \vec{OB} = 6b \quad \vec{BY} = 5a - b$$

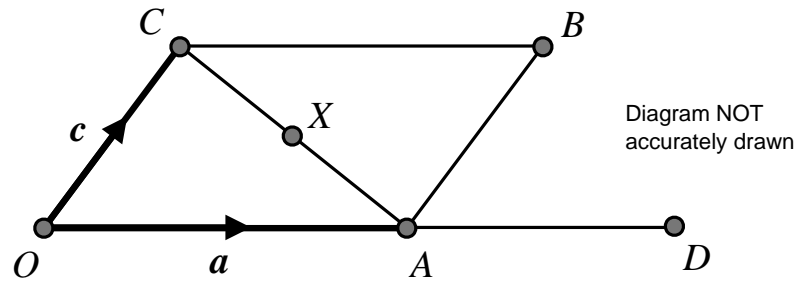
- (i) Use vector algebra to show that  $\vec{AX} = -a + 2b$

[ 2 marks ]

- (ii) Find the ratio  $OX : XY$

[ 4 marks ]

**Question 3**



$OABC$  is a parallelogram with  $\vec{OA} = \mathbf{a}$  and  $\vec{OC} = \mathbf{c}$

$X$  is the midpoint of the line  $CA$ .

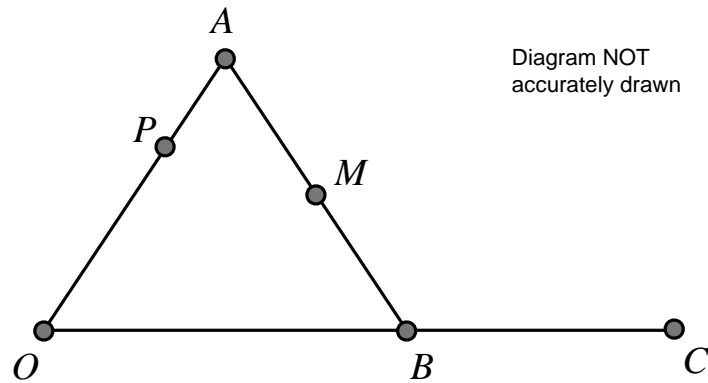
$OAD$  is a straight line.

Given that  $\vec{XD} = 3\mathbf{a} - \frac{1}{2}\mathbf{c}$  find the ratio  $OA : AD$

[ 4 marks ]

**Question 4**

GCSE Examination Question from January 2016, Paper 3H, Q23



$OAB$  is a triangle

$P$  is the point on  $OA$  such that  $OP : PA = 2 : 1$

$C$  is the point such that  $B$  is the midpoint of  $OC$

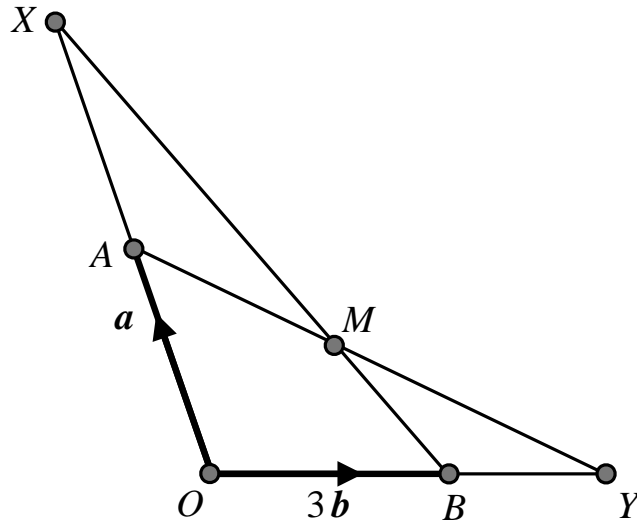
$M$  is the midpoint of  $AB$

$$\vec{OA} = 6\mathbf{a} \quad \vec{OB} = 4\mathbf{b}$$

Show that  $PMC$  is a straight line

[ 5 marks ]

**Question 5**



In the diagram  $\vec{OA} = \mathbf{a}$  and  $\vec{OB} = 3\mathbf{b}$

$OA : AX = 1 : 1$ ,  $OB : BY = 3 : 1$  and  $BM : MX = 1 : 4$

(i) Show that  $\vec{BM} = \frac{2}{5}\mathbf{a} - \frac{3}{5}\mathbf{b}$

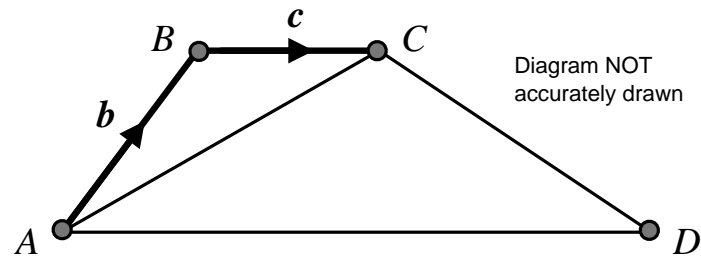
[ 2 marks ]

(ii) Use a vector method to find  $YM : MA$   
Show your working clearly.

[ 4 marks ]

**Question 6**

GCSE Examination Question from January 2017, Paper 3HR, Q23



The diagram shows trapezium  $ABCD$

$BC$  is parallel to  $AD$ ,  $AD = 3BC$ ,  $\vec{AB} = \mathbf{b}$  and  $\vec{BC} = \mathbf{c}$

- (a) Find, in terms of  $\mathbf{b}$  and  $\mathbf{c}$ , the vector  $\vec{CD}$   
Give your answer in its simplest form.

[ 2 marks ]

The point  $P$  lies on the line  $AC$  such that  $AP : PC = 2 : 1$

- (b) Is  $BPD$  a straight line ?  
Show your working clearly.

[ 4 marks ]