

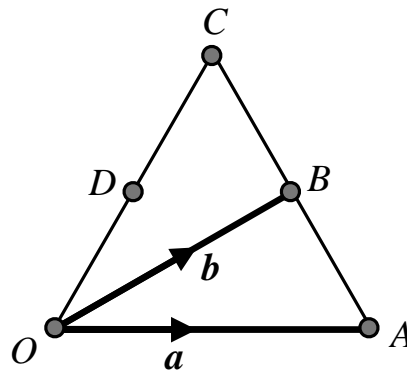
**5.1 Vector Algebra**

The more difficult GCSE vector questions can't be done directly from the provided diagram. Do use the diagram but also work with and trust your vector algebra.

**5.2 Example**

The diagram, which is not drawn to scale, shows an equilateral triangle  $OAC$ . The point  $B$  is the mid-point of  $AC$  and the point  $D$  is the mid-point of  $OC$ .

Furthermore  $\vec{OA} = \mathbf{a}$  and  $\vec{OB} = \mathbf{b}$



Express the following vectors in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ;

(i)  $\vec{AB} =$

(ii)  $\vec{BC} =$

(iii)  $\vec{OC} =$

(iv)  $\vec{OD} =$

(v)  $\vec{DB} =$

(vi) Given that  $\vec{DB} = k \vec{OA}$   
state the value of  $k$

[ 6 marks ]

Teaching Video : <http://www.NumberWonder.co.uk/v9009/5a.mp4> (Part 1)  
<http://www.NumberWonder.co.uk/v9009/5b.mp4> (Part 2)



&lt;= Part 1

Part 2 =&gt;



Complete the example above after watch the teaching videos.

### 5.3 Exercise

*Any solution based entirely on graphical or numerical methods is not acceptable.*

*Make the method used clear.*

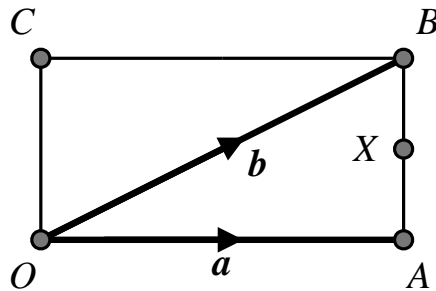
Marks available : 50

#### Question 1

The diagram, which is not drawn to scale, shows a rectangle  $OABC$  with

$$\vec{OA} = \mathbf{a} \text{ and } \vec{OB} = \mathbf{b}$$

The point  $X$  is the mid-point of  $AB$ .



(a) Express the following vectors in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ;

(i)  $\vec{CB} =$

(ii)  $\vec{AB} =$

(iii)  $\vec{AX} =$

[ 3 marks ]

(b) Work out  $\vec{CX}$  by using the path.

(i)  $\vec{CX} = \vec{CB} - \frac{1}{2} \vec{AB}$

[ 2 marks ]

(ii)  $\vec{CX} = \vec{CB} - \vec{OB} + \vec{OA} + \vec{AX}$

[ 2 marks ]

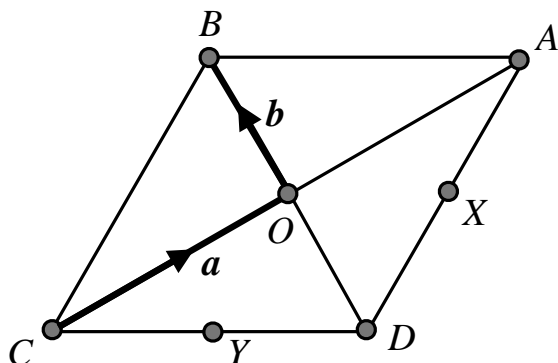
### Question 2

The diagram, which is not drawn to scale, shows a rhombus  $ABCD$ .

The two diagonals of the rhombus intersect at  $O$ .

The point  $X$  is the mid-point of  $AD$  and the point  $Y$  is the mid-point of  $CD$ .

Furthermore,  $\vec{CO} = \mathbf{a}$  and  $\vec{OB} = \mathbf{b}$



(a) Express the following vectors in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ;

(i)  $\vec{DA} =$

(ii)  $\vec{DX} =$

(iii)  $\vec{CD} =$

(iv)  $\vec{YX} =$

[ 1, 1, 1, 1 mark ]

(b) Given that  $\vec{YX} = k \vec{CA}$  state the value of  $k$

[ 1 mark ]

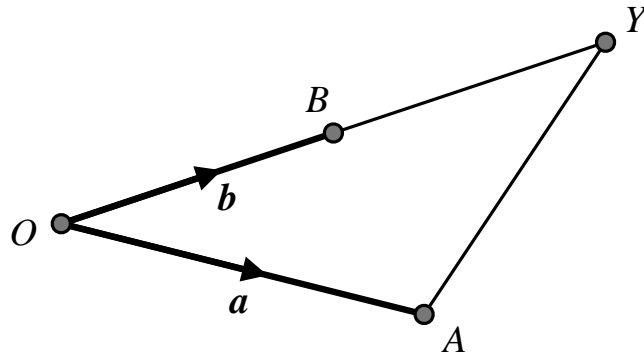
(c) If  $|\mathbf{a}| = 15$  and  $|\mathbf{b}| = 7$ , determine  $|\mathbf{a} + \mathbf{b}|$  using the fact that the diagonals of a rhombus are mutually perpendicular.

[ 1 mark ]

**Question 3**

The diagram, which is not drawn to scale, shows a triangle  $OAY$ .  
The point  $B$  is the mid-point of  $OY$ .

Furthermore,  $\vec{OA} = \mathbf{a}$  and  $\vec{OB} = \mathbf{b}$



(a) Express the following vectors in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ;

(i)  $\vec{OY} =$                       (ii)  $\vec{AY} =$

[ 2 marks ]

$X$  is the mid-point of  $OA$

(b) Write down, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , an expression for  $\vec{XB}$

[ 1 mark ]

(c) Show that  $\vec{XB}$  is parallel to  $\vec{AY}$ , by writing a relationship between them of the form  $\vec{XB} = k\vec{AY}$

[ 1 mark ]

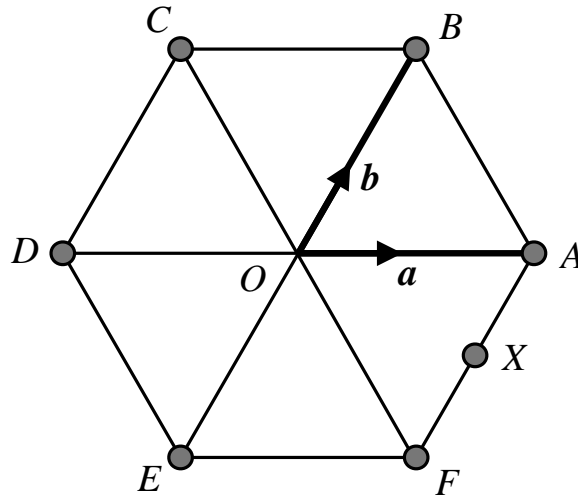
(d) If  $\vec{OA}$  and  $\vec{AY}$  are mutually perpendicular and  $|\mathbf{a}| = |\mathbf{b}| = 3$  cm what is  $|\vec{AY}|$  ?

[ 1 mark ]

**Question 4**

The diagram, which is not drawn to scale, shows a regular hexagon  $ABCDEF$ .  
The spokes of the hexagon intersect at  $O$ .

Furthermore,  $\vec{OA} = \mathbf{a}$  and  $\vec{OB} = \mathbf{b}$



(a) Express the following vectors in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ;

(i)  $\vec{DB} =$

(ii)  $\vec{DC} =$

(iii)  $\vec{FC} =$

(iv)  $\vec{FD} =$

[ 4 marks ]

The point  $X$  is the mid-point of  $FA$ .

(b) Write down, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , an expression for  $\vec{DX}$

[ 1 mark ]

(c) If the hexagon has sides of length 4.3 cm, what is  $|\vec{DB}|$  ?  
HINT : The Cosine Rule

[ 2 marks ]

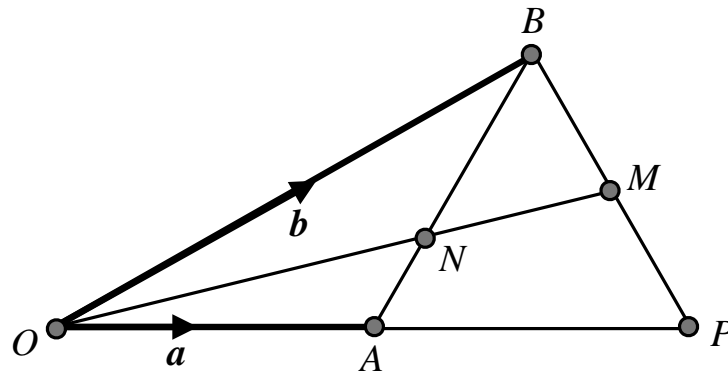
**Question 5**

The diagram, which is not drawn to scale, shows an equilateral triangle  $APB$  and an isosceles triangle,  $OAB$ , where  $|\vec{OA}| = |\vec{AB}|$

The point  $M$  is the mid-point of  $PB$ .

$$\vec{AN} = \frac{1}{3} \vec{AB}$$

Furthermore,  $\vec{OA} = \mathbf{a}$  and  $\vec{OB} = \mathbf{b}$



(a) Express the following vectors in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ;

(i)  $\vec{OP} =$                       (ii)  $\vec{PB} =$

(iii)  $\vec{OM} =$                       (iv)  $\vec{AN} =$

(v)  $\vec{ON} =$

[ 1, 1, 1, 1, 2 marks ]

(b) Given that  $\vec{OM} = k \vec{ON}$  find  $k$ .

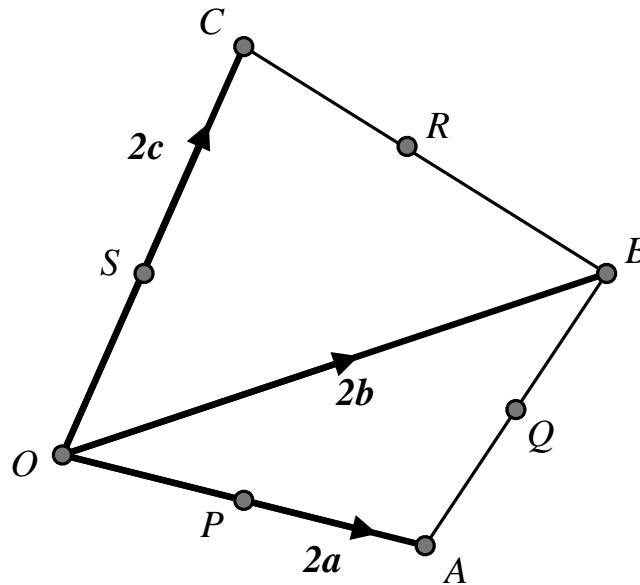
[ 2 marks ]

**Question 6**

The diagram, which is not drawn to scale, shows a quadrilateral  $OABC$

in which,  $|\vec{OA}| = 2a$ ,  $|\vec{OB}| = 2b$  and  $|\vec{OC}| = 2c$

Points  $P$ ,  $Q$ ,  $R$  and  $S$  are the midpoints of the sides  $OA$ ,  $AB$ ,  $BC$  and  $CO$  respectively.



(a) Express the following vectors in terms of  $a$ ,  $b$  and  $c$ ;

(i)  $\vec{AB} =$

(ii)  $\vec{BC} =$

(iii)  $\vec{PQ} =$

(iv)  $\vec{QR} =$

(v)  $\vec{PS} =$

[ 1, 1, 1, 2, 1 marks ]

(b) Describe the relationship between  $\vec{QR}$  and  $\vec{PS}$

[ 1 mark ]

(c) What sort of quadrilateral is  $PQRS$ ?

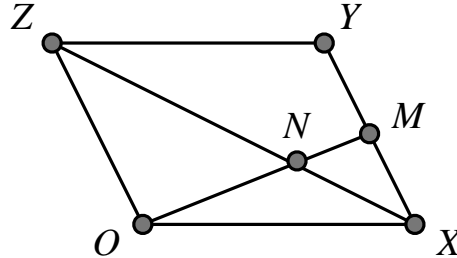
[ 1 mark ]

### Question 7

In the diagram,  $OXYZ$  is a parallelogram.

$M$  is the mid-point of  $\overrightarrow{XY}$

Furthermore,  $\overrightarrow{OX} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$  and  $\overrightarrow{OZ} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$



- (i) Write down the vectors  $\overrightarrow{XM}$  and  $\overrightarrow{XZ}$

[ 2 marks ]

- (ii) Given that  $\overrightarrow{ON} = v \overrightarrow{OM}$  write down in terms of  $v$  the vector  $\overrightarrow{ON}$

[ 2 marks ]

- (iii) Given that  $\overrightarrow{ON} = \overrightarrow{OX} + w \overrightarrow{XZ}$  find in terms of  $w$  the vector  $\overrightarrow{ON}$

[ 1 mark ]

- (iv) Solve two simultaneous equations to find  $v$  and  $w$

[ 3 marks ]

- (v) Explain the significance of your solution.

[ 1 mark ]