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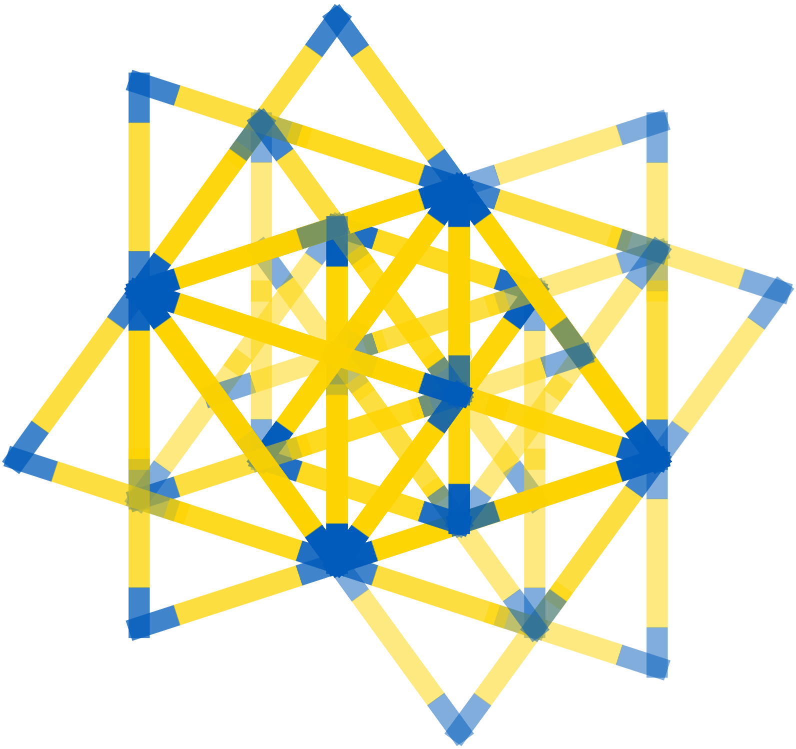


Remembering Professor Uwe Grimm

Martin Hansen

It was Uwe who introduced me to that most fascinating of mathematical toys, the infinite Fibonacci word. It's a deceptively simple substitution, θ, on an alphabet of only two letters, $\mathcal{A}(a, b)$, defined by $a \to ab$ and $b \to a$. It gives us the finite Fibonacci words, $\mathcal{F}_n = \theta^n(a)$. The first few are

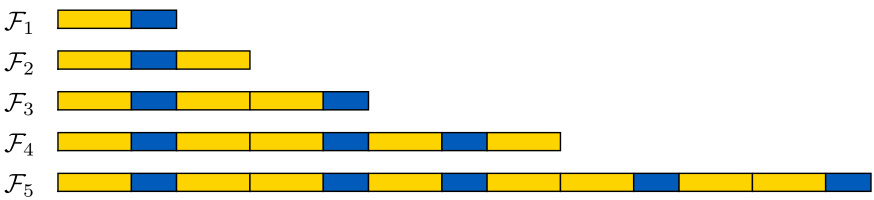
$$\mathcal{F}_0 = a, \quad \mathcal{F}_1 = ab, \quad \mathcal{F}_2 = aba, \quad \mathcal{F}_3 = abaab$$

and so on. Throw away the last couple of letters on any given word and what's left is a palindrome. As example, $\mathcal{F}_4 = abaababa$ which, without its rightmost two letters, is $abaaba$. This palindromic nature, along with the remarkable concatenation property that

$$\mathcal{F}_n \;=\; \mathcal{F}_{n-1}\mathcal{F}_{n-2} \;\text{ for } n \geq 3,$$

guarantees that the Fibonacci words abound with symmetries. As $n \to \infty$ the infinite Fibonacci word emerges as a fixed point of the iteration.

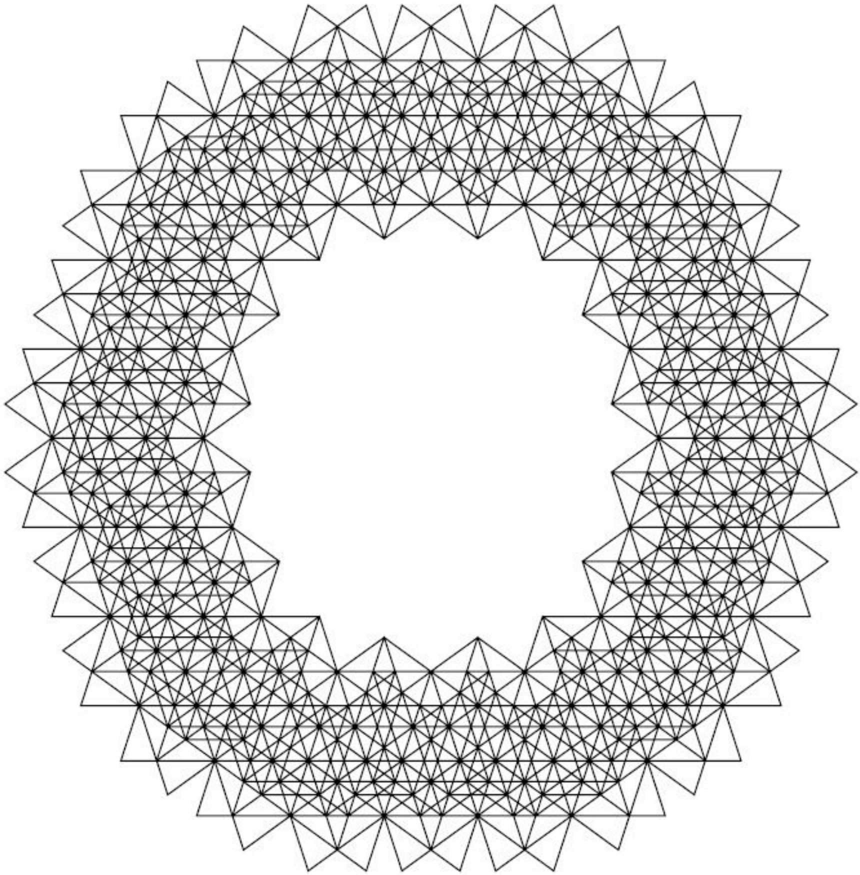
Uwe took pleasure in finding geometric visualisations to complement his algebraic researches. These were often stunningly beautiful creations that non-mathematicians could marvel over. When he died, I had just begun studying the Open University's M840 graduate course, *Aperiodic Tilings and Symbolic Dynamics*. In the course topic guide (co-authored with Reem Yassawi), he showed how, via the Fibonacci word's substitution incidence matrix, the left eigenvector gave rise to an aperiodic tiling of $\mathbb{R}^+$.



For my dissertation I ended up running with this idea, under the watchful eye of Dan Rust, who kindly stepped in to supervise Uwe's orphaned students and keep the course going. I took Uwe's tiled path and twisted it back and forth, often with it tiling over itself, and looked at the properties of the resulting figures. The twisting was via a drawing rule that took each letter of the Fibonacci word in turn and used it as an instruction to say how the next tile should be placed. Some attractive images resulted. To give a flavour of what can occur the adjacent image is for $\mathcal{F}_{22}$ under the following drawing rule.

Symbol	Action
a	forward ϕ (The golden ratio, about 1.618)
b	forward 0.5, turn 108°, forward 0.5.

I like to think that my visualisation of $\mathcal{F}_{22}$ is in the spirit of the mathematics that inspired it; Uwe's mathematics. And that he would approve.



Contents

03.00

Problem 309.8 – Pythagorean triples

Find all positive integers n such that there exist positive integers a and b such that

$$n^2 \;=\; (n-1)^2 + a^2 \;=\; (n-2)^2 + b^2.$$

Is it possible to find positive integers a, b and c such that

$$n^2 \;=\; (n-1)^2 + a^2 \;=\; (n-2)^2 + b^2 \;=\; (n-3)^2 + c^2?$$

Front cover Fibonacci word $\mathcal{F}_{12}$; see page 6.