2019

THE ARNOLD HAGGER MATHEMATICS PRIZE COMPETITION

3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679821480865132823066470938446095505822317253594081284811174502

THURSDAY 31st January

7.15pm - 8.45pm

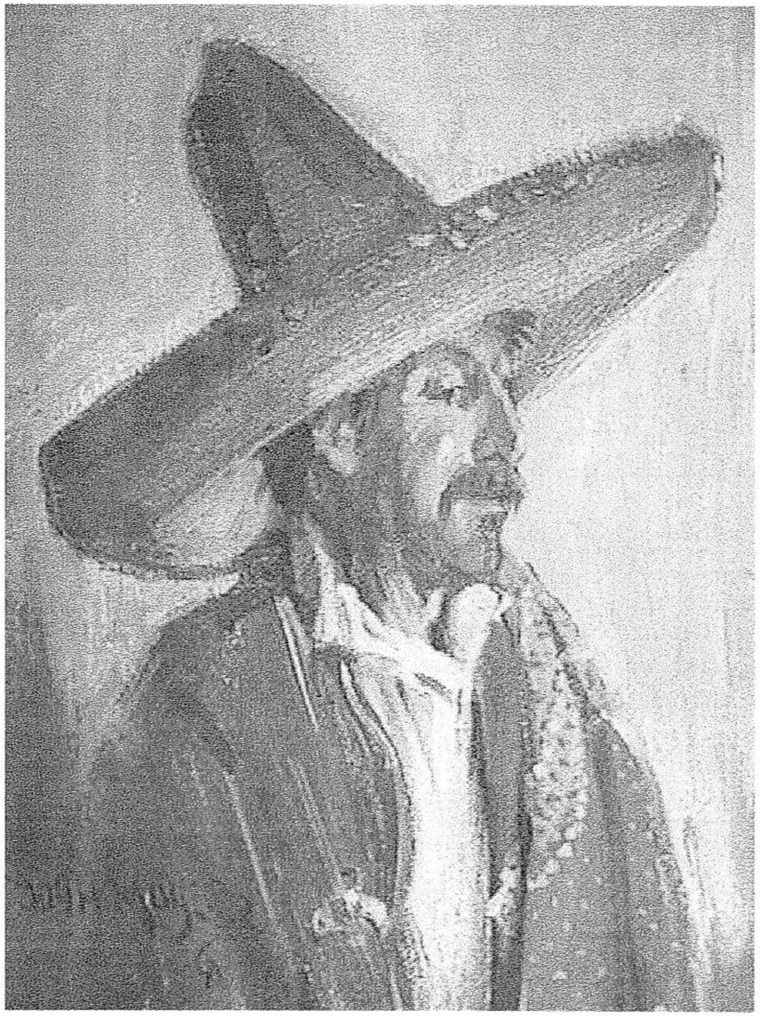
At Shrewsbury School

Calculators MAY be used

INTUS SI RECTE NE LABORA

The Arnold Hagger Mathematics Prize Competition 2019

* The thirteen questions may be answered in any order.
* Make your methods of solution clear by including all working and reasoning.
* The marks allocated to each question is shown - either **[5]** or **[10]** marks.
* **Calculators MAY be used**.



Question One : **B·A·N·D·I·T·S**

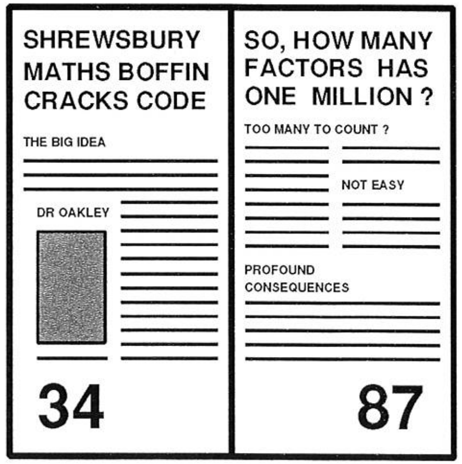
[5] A normal bandit has two arms.
A one armed bandit has one arm.
A harmless (armless) bandit has no arms.

At a bandit stronghold there are 243 bandits with a total of 269 arms.
The total number of normal bandits and one armed bandits is twice the number of harmless bandits.

How many one armed bandits are there ?

Question Two : **S·U·M O·F P·A·G·E·S**

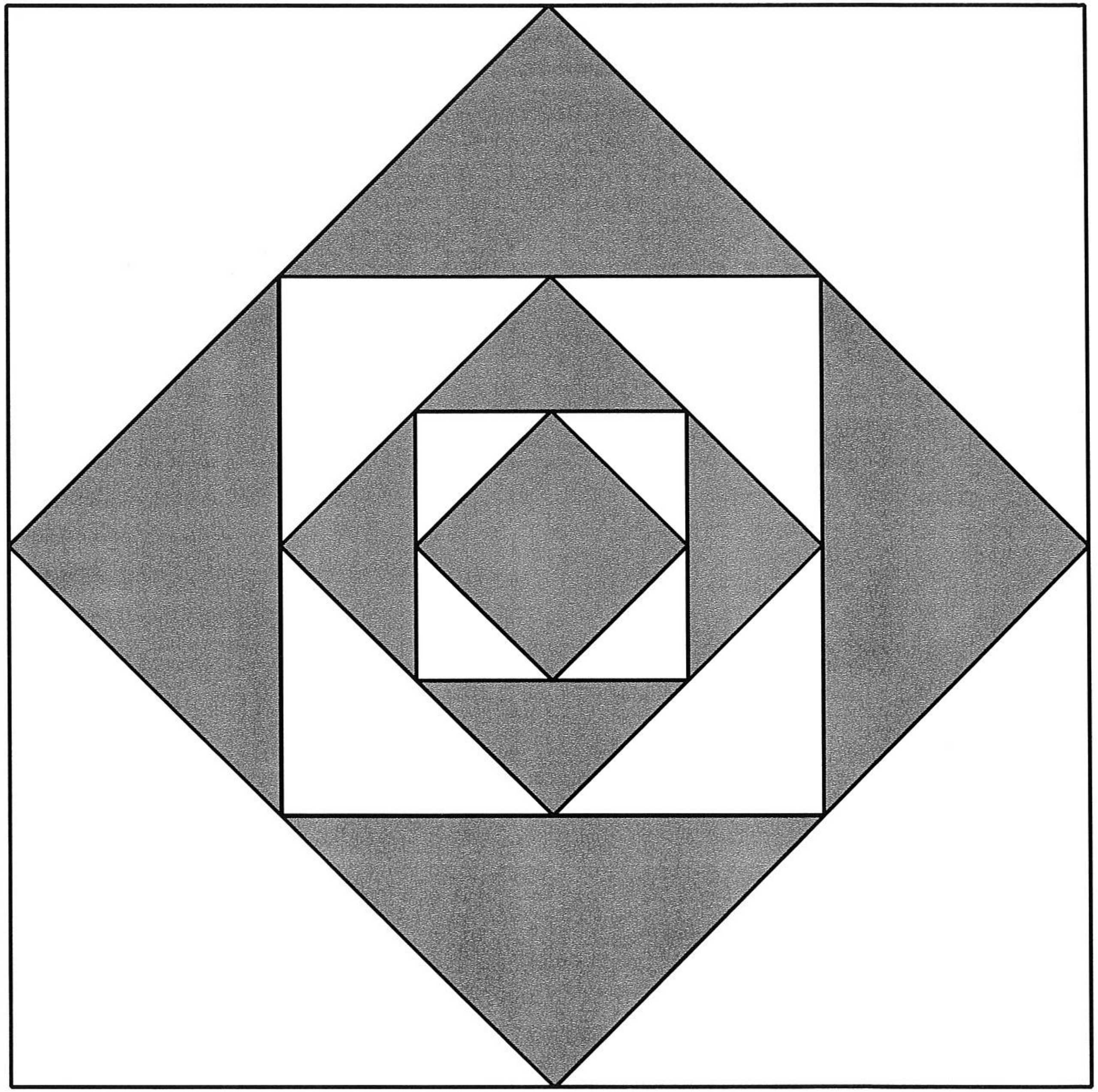
[5] The middle section of a magazine has been removed to reveal page 34 opposite page 87.
If I had previously added up all the page numbers in the magazine, what would my total have been ?



Question Three : **S·Q·U·A·R·E·D**

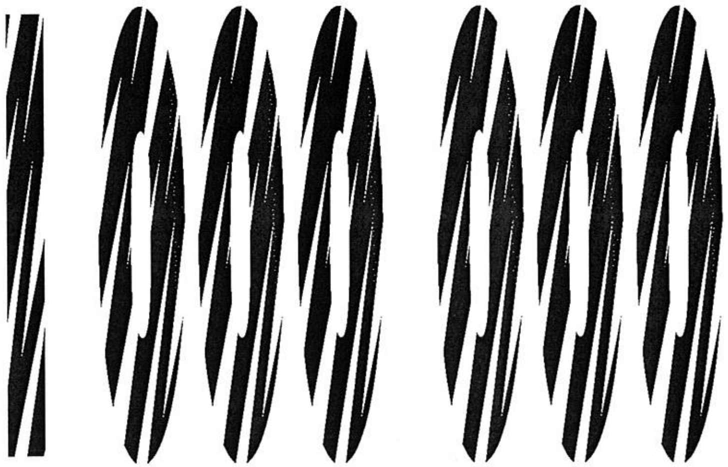
[5] To the right I've drawn a square inside a square inside a square inside a square inside a square inside a square.

What fraction of the largest square is shaded ?



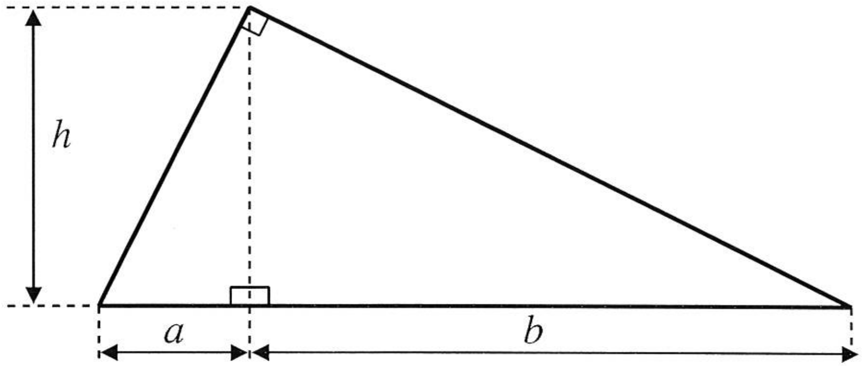
Question Four : **F·A·C·T·O·R·S O·F O·N·E M·I·L·L·I·O·N**

[10] How many factors does the number one million have ?



Question Five : **A M·E·A·N T·R·I·A·N·G·L·E**

[5] A right angled triangle can be thought of as having its hypotenuse as its base and a perpendicular height, h. That perpendicular height divides the base into two lengths, a and b as shown below.



Find an elegant expression for h in terms of a and b.

Question Six : **W·H·E·N N·I·N·E + T·W·O = F·I·V·E**

[5] Each letter represents a different digit between 0 and 9. The two numbers being added are each even. If $W = 4$, and what are the values of the other letters ?

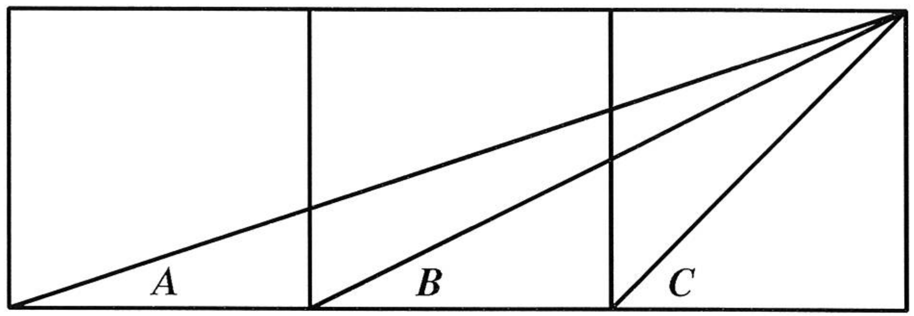
```
    N I N E
+     T W O
-----------
    F I V E
```

Question Seven : **B·E·S·T O·F T·H·R·E·E**

[10] How many three-digit numbers formed only of odd digits are divisible by 3 ?

Question Eight : **S·I·M·P·L·E P·R·O·O·F S·O·U·G·H·T**

[5] Prove in as simple a way as you can that in the diagram below of three squares, angle A plus angle B equals angle C



Question Nine : **M·A·T·H·E·M·A·T·I·C·S A·W·A·R·D**

[10] A teacher of mathematics wishes to give out £7 and £11 awards. To receive an award a pupil must first work out the largest amount of integer pounds that cannot be made using just these two types of award. What is that amount ?

Question Ten : **m & n**

[10] In a box are 12 beads, m of which are black, the rest white.
Arnold takes one bead at random from the box.
He places it into a second box which already contained 8 beads, n of which were black, the rest white.
Arnold now takes one bead at random from this second box.

Given that the probability that Arnold takes 2 black beads is $\frac{2}{6}$

and that the probability that Arnold takes 2 white beads is $\frac{1}{6}$

determine the values of m and n

Question Eleven : **T·W·I·N T·W·I·N P·R·I·M·E·S**

[10] The twin primes less than 100 are (3, 5), (5, 7), (11, 13), (17, 19) (29, 31) (41, 43) (59, 61) and (71, 73)
It is almost certain that there are an infinite number of twin primes but no one to date has managed to prove this.

(i) Let three consecutive integers be n-1, n and n+1
If n-1 and n+1 are both prime explain why, for $n > 4$, n must be divisible by 6

	2	3		5		7			
11		13				17		19	
		23						29	
31						37			
41		43				47			
		53						59	
61						67			
71		73						79	
		83						89	
						97			
101		103				107		109	

Primes $\leqslant 110$

Twin twin primes are harder to find but the first in the list is (5, 7, 11, 13) and the fourth in the list is (821, 823, 827, 829).

(ii) With the help of the prime number grid, give the second and third twin twin primes.

(iii) Let nine consecutive integers be n-4, n-3, n-2, n-1, n, n+1, n+2, n+3, n+4
If n-4, n-2, n+2 and n+4 are all prime for $n > 7$, what is the largest integer that n must be divisible by ?
Prove your answer.

Question Twelve : **R·E·P·L·I·C·A·N·T B·I·S·E·C·T·I·O·N**

[10] For any two digit number, *ab*, it's six digit replicant is *ababab*

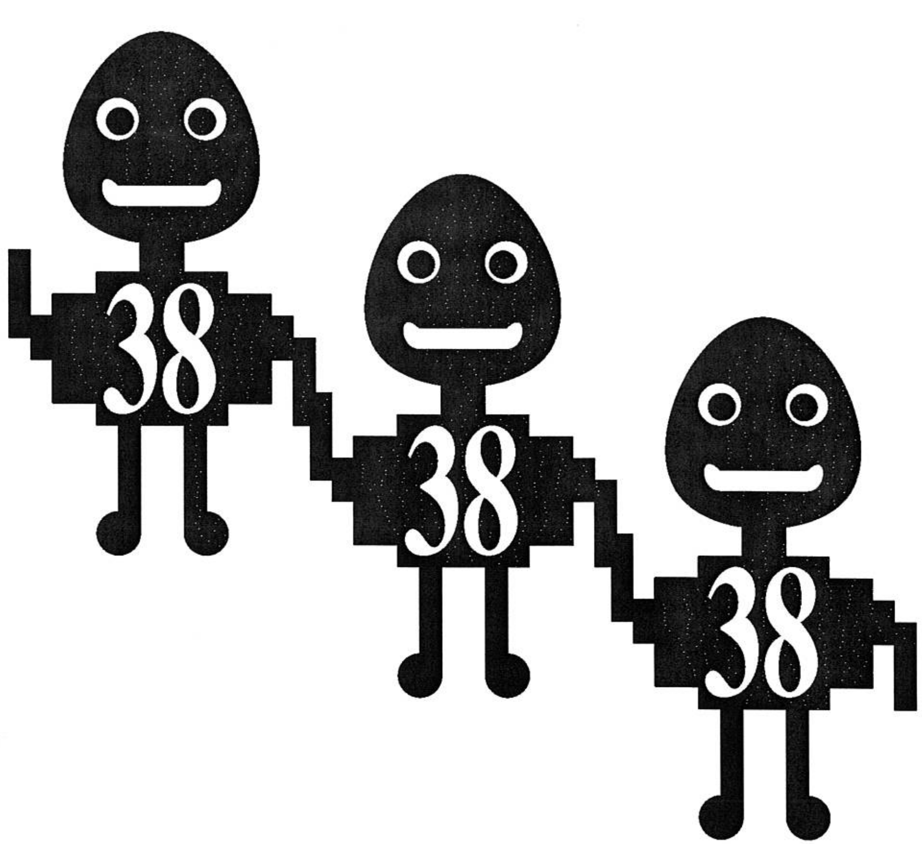
For example, 38 gives the six digit replicant 383838

(i) Prove that all six digit replicants divide by 10101

A bisected six digit replicant is of the form $(aba)^2 - (bab)^2$

For example 38 gives the bisected six digit replicant $383^2 - 838^2$

(ii) Prove that all bisected six digit replicants are also divisible by 10101



Question Thirteen : **C·O·I·N·C·I·D·E·N·T·A·L·L·Y**

[10] Jenny multiplies two integers which differ by 13.

Mark multiplies two integers which differ by 4.

Coincidentally, they obtain the same product *T*.

Determine all possible values of *T*.

END OF PAPER

Martin Hansen, January 2019

With thanks to Ian Payne for his enthusiastic support for the competition
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and to Sara Lunzy for producing a magnificent poster for the event