Name	Set

Arnold Hagger Competition

January 2017

This paper carries a total of 100 marks.

Answer as many questions as you can. Do not spend too long on any one question, if you become stuck.

Answers without working or explanation will receive few, if any marks. Short, elegant solutions will gain more marks than long, contorted ones.

Should you require more space to answer a question, use a separate piece of paper for each question and insert it in the correct place within this booklet.

You **may** use a calculator in this paper.

1) Which number gives the same result when it is added to $5\frac{1}{3}$ as when it is multiplied by $5\frac{1}{3}$?

[3 marks]

2) A gymnast did 200 back flips in 5 days, each day doing 4 more than on the previous day. How many did she do on the first day?

[3 marks]

3) Copy this equation and draw one line on it to make it correct:

$$5 + 5 + 5 + 5 = 555$$

[1 mark]

4) A man has 29 socks in his drawer: 9 identical blue, 8 identical grey and 12 identical black.
The lights have fused and he is completely in the dark.
How many socks must he take out to make certain that he has a pair of each colour?

[3 marks]

5) Calculate $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4} \times 1\frac{1}{5} \times 1\frac{1}{6} \times \ldots \times 1\frac{1}{2017}$

[3 marks]

6) There is to be a tennis tournament with 1001 players.
You are the organiser and it is a simple knockout competition. Two people play tennis and the winner goes through to the next round.
i.e. In round one there will be 500 matches, the remaining one person will get a 'by' into the next round.

How many matches need to be played to find the winner?

[2 marks]

7) The first six triangular numbers are: 1, 3, 6, 10, 15, 21, ...

What is the next triangular number after 1275?

[5 marks]

8) Without a calculator, **show** (numerically or algebraically) how to determine which of the following expressions has a greater value: $10^{1/10}$ or $3^{1/3}$.

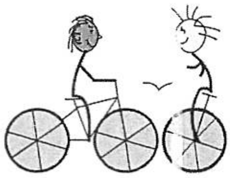
[5 marks]

9) There is a school with 1,000 students and 1,000 lockers. On the first day of term the headteacher asks the first student to go along and open every single locker, he asks the second to go to every second locker and close it, the third to go to every third locker and close it if it is open or open it if it is closed, the fourth to go to the fourth locker and so on. The process is completed with the thousandth student. How many lockers are open at the end?

[5 marks]

10) Two cyclists are racing towards each other.
They start 100 metres apart. The unicycle is moving at 2 metres per second and the bicycle at 3 metres per second.

There is a bird flying at 4 metres per second. It flies from the bicycle to the unicycle then back to the bicycle, then returns and so on and on…



Question: How far does the bird fly in total?

[5 marks]

11) What is the largest odd number that is a factor of $\dfrac{2014 \times 2015 \times 2016}{2013 - 2014 + 2015 - 2016 + 2017}$?

[5 marks]

12) When the positive integers are arranged in order, filling in the successive diagonals of an *infinite grid* from top to bottom, as shown, the integer 41 is in the (5, 5) spot. What integer would we see in the (10, 10) spot if the rest of the grid were visible?

1	2	4	7	11	
3	5	8	12	17	
6	9	13	18	24	...
10	14	19	25	32	
15	20	26	33	41	
		⋮			

[5 marks]

13) The equation $8^x + 4 = 4^x + 2^{x+2}$ has:

a) no real solutions

b) one real solution

c) two real solutions

d) three real solutions

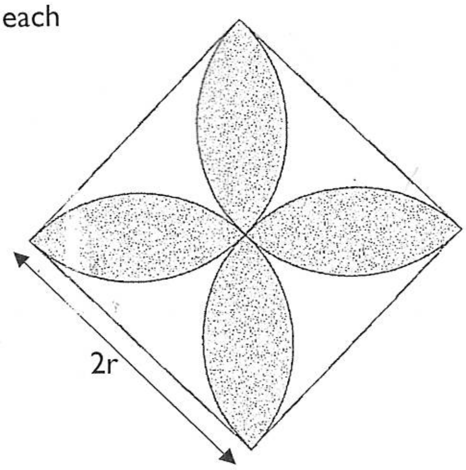
Explain your answer.

[5 marks]

14) Four semicircles of radius r are drawn inside a square of side 2r so that each semicircle has a side of the square as its diameter.

The four-leaf figure formed where the semicircles overlap inside the square is shown shaded.

Find the area of the shaded figure in terms of r.



[5 marks]

15) Solve $\dfrac{1}{(x+1)^2}+\dfrac{1}{(x-1)^2}=1$ giving the answers exactly.

[10 marks]

16)

Mustapha and Ali were camel traders, in days of yore, who decided to sell their stock and become sheep traders. They took their camels to market and received for each camel a number of dinars, equal to the total number of camels sold. With this money they purchased sheep at 10 dinars each, and with the money left over they purchased a goat.

On the way home they argued, so decided to share the sheep and found that they had one left over. So Ali kept that sheep and gave Mustapha the goat. "But I have less than you" said Mustapha, "because a goat is worth less than a sheep."

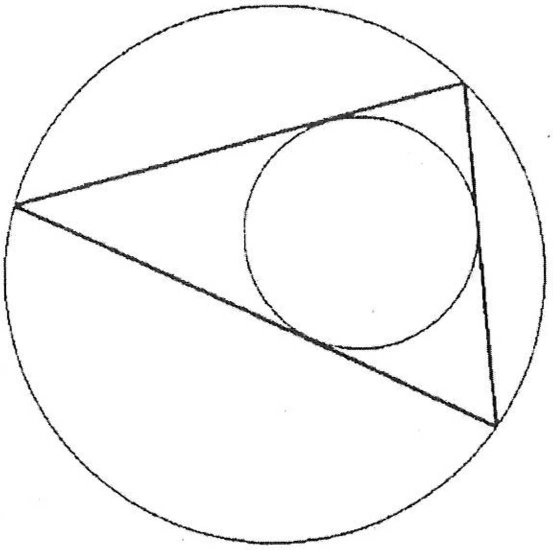
"All right" said Ali, "I will give you one of my wives to make up the difference."

Question: what was the value of the wife?

[10 marks]

17) Triangle ABC has sides of length *a, b, c*. The circumcircle of the triangle has radius *R* and the incircle has radius *r*. The semi-perimeter *s* of the triangle is defined to be $s = \frac{1}{2}(a + b + c)$.

Prove that $abc = 4Rrs$



Hints:

1) *Consider the area of the triangle in two different ways.*

2) *You may use the full sine rule:*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

[10 marks]

18) A Delectable Number is a nine-digit number that:

a) Contains the digits 1 – 9 exactly, once each (no zero), and

b) The numbers created by taking the first n digits (n runs from 1 – 9) are each divisible by n, so the first digit is divisible by 1 (it always will be), the first two digits form a number divisible by 2, the first three digits form a number divisible by 3, and so on.

For example, taking the nine-digit number 123456789:

$$1 \div 1 = 1$$

$$12 \div 2 = 6$$

$$123 \div 3 = 41$$

However, this then breaks down because 1234 is not divisible by 4.

There is only one nine-digit number that meets the required conditions. Can you find it?

[15 marks]