THE ARNOLD HAGGER MATHEMATICS PRIZE COMPETITION

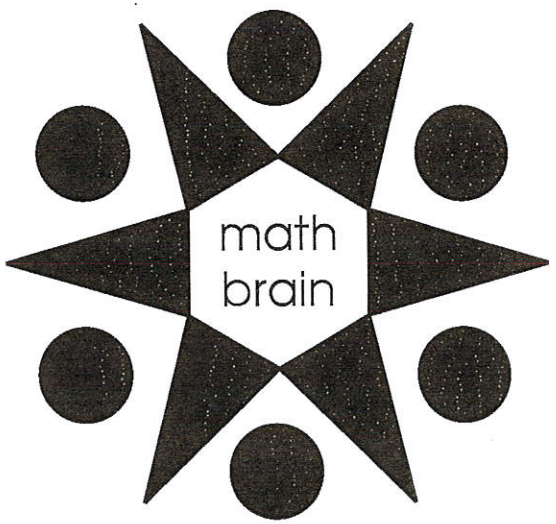
2009

WEDNESDAY 21st January

7.15pm - 8.45pm

Room M7

Calculators MAY be used



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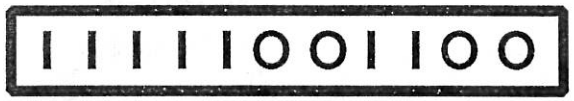
The Arnold Hagger Mathematics Prize Competition 2009

* The twelve questions may be answered in any order.
* Make your methods of solution clear by including all working and reasoning.
* The marks allocated to each question is shown - either **[5]**, **[10]** or **[15]** marks.
* **Calculators MAY be used**.

Question One : **C·R·A·I·G**

[5] Above the entrance to the ***Craig Building*** is the following etching:



Explain the etching.

Question Two : **R·E·A·D M·Y M·I·N·D**

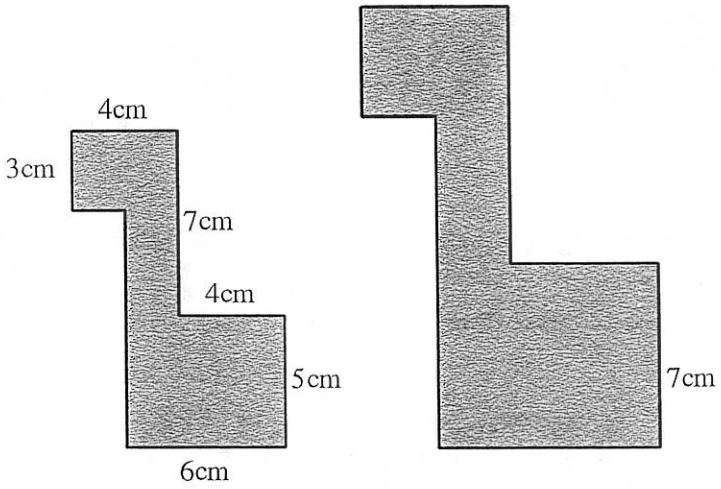
[5] In my mind is a ***square*** number, less than 1000, which has remainder one if I divide it by 2, 3, 5 or 7.
What is the number that I have in mind ?

Question Three : **A G·R·E·A·T E·I·G·H·T**

[5] State the units digit of;

Question Four : **L·I·T·T·L·E A·N·D L·A·R·G·E**

[5] The two shapes shown below are similar.
What is the area of the larger shape ?



Question Five : **S·O·C·K I·T T·O M·E**

[5] In my sock box are sixteen socks, some red, some green.
On randomly pulling out first one, then a second, (without replacing the first) the probability they're both green is 1:12.
How many of my sixteen socks are red ?

Question Six : **M·I·N·D E·S·C·A·P·E**

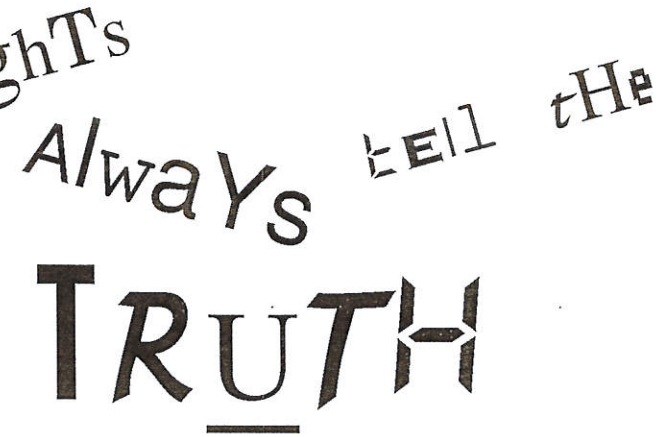
[5] You are shipwrecked on an island inhabited by two tribes; Knights (who always tell the truth), and Knaves (who always lie). When captured you are looked after by two friendly jailers who are called Tweedledum and Tweedledee.
Island law states that you can regain your freedom if you discover to which tribes your two jailers belong.
To help you, Tweedledum says,

"We come from different tribes".

But Tweedledee immediately says

"Oh no we don't !".

What can you conclude about your two jailers ?



Question Seven : **M·U·L·T·I·T·R·A·C·K M·I·N·D**

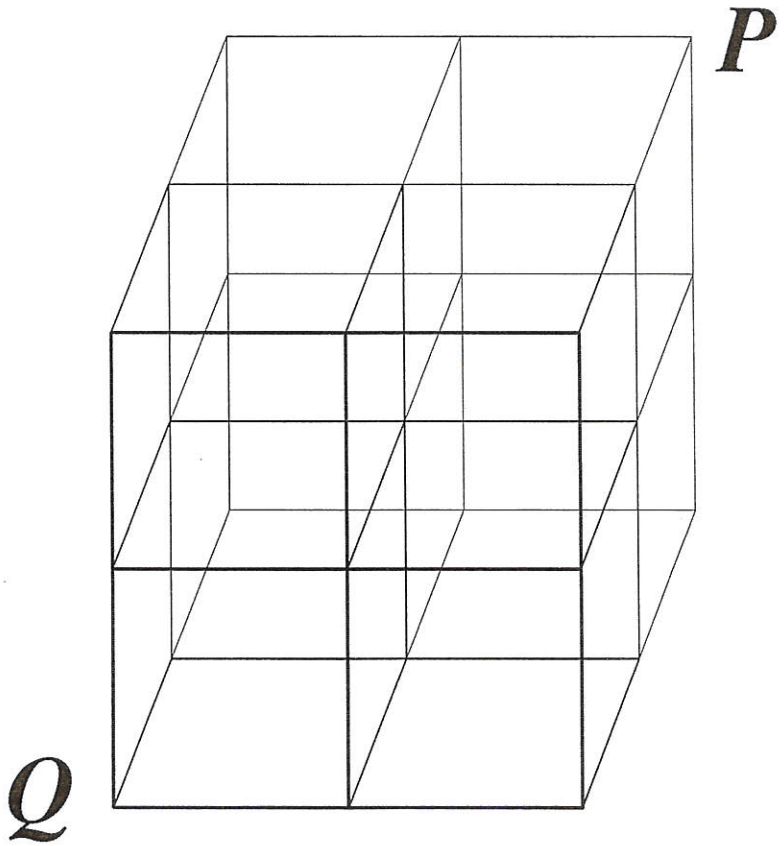
[10] Armed with drinking straws and thread, Mr Hansen's set have been constructing wire frame geometric models. .

One such model is shown, made from 54 straws.

A spider is at vertex *P* and a fly is stuck at vertex *Q*.

Starting from *P*, paths can be travelled to *Q* along the straws.

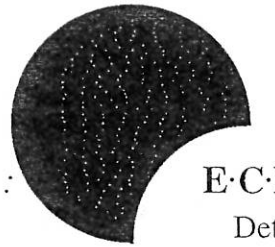
If each movement along a path takes the spider closer to the fly, how many possible paths are there from *P* to *Q* ?



Question Eight : **W·E·L·C·O·M·E T·O T·H·E A·L·G·A·E B·A·R**

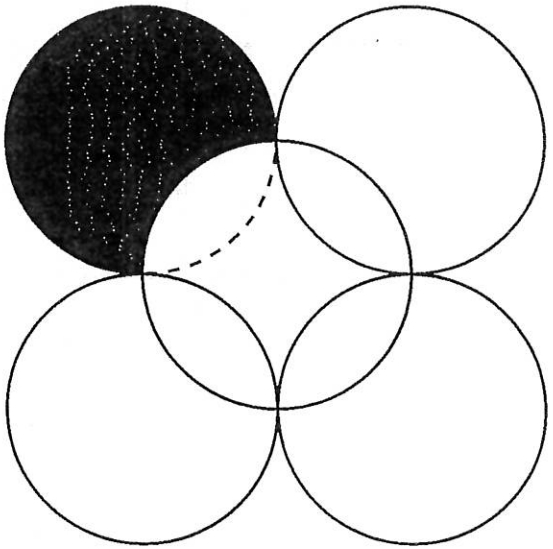
[10] Let $x + y = A$ and $x^2 + y^2 = B$.

Find, in as simple a form as possible, an expression for $x^3 - y^3$ in terms of A and B.

Question Nine : 

E·C·L·I·P·S·E

[5] Determine the area of the shaded shape which was constructed from five identical circles each of radius "*R*" arranged as shown to the right.



Question Ten : **P·R·I·M·E R·E·S·U·L·T**

[15] Prove that

$$p^2 \bmod 2p = p$$

where p is any odd prime number.

NOTE : The "mod" function means "has a remainder upon division by".

EXAMPLES : "13 mod 5 = 3" means "13 has a remainder upon division by 5 of 3".

"13 mod 6 = 1" means "13 has a remainder upon division by 6 of 1".

"13 mod 7 = 6" means "13 has a remainder upon division by 7 of 6".

Question Eleven : **A·S E·A·S·Y A·S A B C**

[15] a, b, and c are positive real numbers with the property that;

$$ab + bc + ca = 1$$

Show that; $a + b + c \geqslant \sqrt{3}$

Question Twelve : **A·I·M·I·N·G F·O·R P·E·R·F·E·C·T·I·O·N**

[15] In this question, n is a natural number.
i.e. $n \in \{ 0, 1, 2, 3, 4, 5, ... \}$

Consider the expression $2 + 2\sqrt{1 + 12n^2}$

Prove that if this expression has an integer value then that value is also a perfect square.

END OF PAPER

Martin Hansen, January 2009.

(with thanks to Charles Oakley, Jerome Armstrong and Martin Cropper for advice and suggestions)