



Shrewsbury School

## **ARNOLD HAGGER PRIZE 2008**

**90 minutes**

*This paper carries a total of 100 marks.*

*Attempt as many questions as you can. Do not spend too long on any one question if you get stuck.*

*You may use a calculator in this paper.*

*You should show full working.*

1.  $47.5 + 3862.7 + 125.6 + 1583.1 = 45085.7$

This sum is wrong, because the decimal points have not been written in correctly – in fact the decimal point is in the correct place in just one of the numbers.

Rewrite the sum so that it makes sense, with the decimal points in the correct places.

(5 marks)

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2. A firefighter, standing on a rung of a ladder, notices that there are twice as many rungs below the rung she is standing on as there are above it.

After going down 8 rungs, she notices that there are the same number of rungs above and below the rung she is now on.

How many rungs are there on the ladder?

(5 marks)

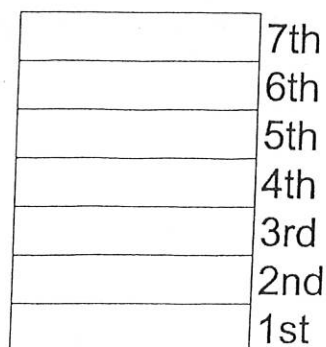
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3. Sort the colours:

- The yellow block is directly below the blue block.
- There is one other colour between the yellow and black blocks.
- The red block is on the bottom.
- The pink block is in the black's position plus the red's position.
- The purple block is above the white.
- The black block is touching the red.

Copy the diagram and fill in the boxes.

(5 marks)



4. After a cyclist has gone two thirds of the distance of his journey he gets a puncture. Finishing on foot he spends twice as long walking as he did riding.

How many times as fast does he ride as walk?

(5 marks)

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5. What is the sum of the digits in the sequence 1, 2, 3, 4, 5, ..., 99, 100?  
(N.B. Sum of the digits, not the numbers.)

(5 marks)

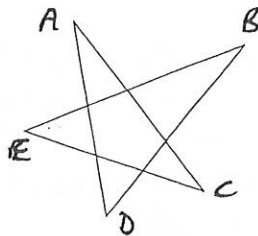
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6. Two integers multiply together to give 1,000,000,000.  
Neither of them contains a zero.  
What are they?

(5 marks)

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7. In a pentagonal star such as the one shown, show that the sum of the angles at the five vertices (A, B, C, D, E) is 180 degrees.



(5 marks)

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8. After a banquet at the Indian restaurant, there were 65 serving dishes to wash up. "How many guests were at the banquet?" asked the manager. "I didn't have time to count them", the waiter replied, "but I do know that there was one dish of rice for every two people, there was a vegetable dish for every three people, and a large curry dish for every four people."

How many guests were there at the banquet?

(7 marks)

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9. The points A, B, C, D are placed consecutively on a circle with diameter AC so that the straight line distances are:

- $AB = 7\text{cm}$
- $BC = 24\text{cm}$
- $CD = 15\text{cm}$

What is the area of the quadrilateral ABCD?

(7 marks)

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10. Find all integers  $n$  such that  $\frac{n+3}{n-1}$  is also an integer.

(7 marks)

11. A right angled triangle has sides of length  $a$ ,  $a + 4$ , and  $2a$ . What are the possible values of  $a$ ?

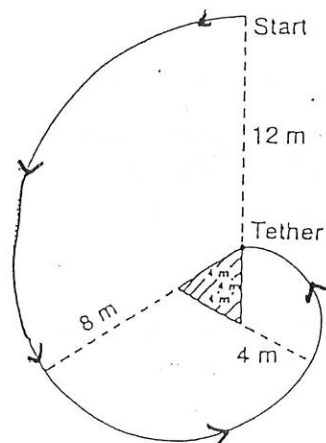
(7 marks)

12. In mathematics there is a notation “!” (factorial) for the product of consecutive integers starting from 1. For example  $1 \times 2 \times 3 \times 4 \times 5$  is written as  $5!$  and its value is 120.

Given that  $n^3$  is a divisor of  $15!$  what is the greatest possible integer value of  $n$ ?

(7 marks)

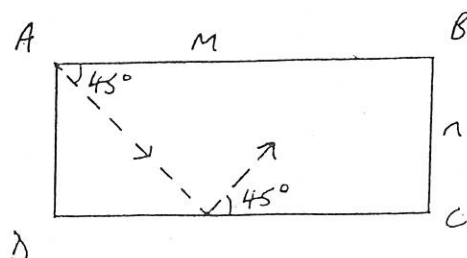
13. A goat is tethered to one corner of a pillar with an equilateral triangle as cross-section, of side 4 metres. The rope is 12 metres long and initially follows the line of one of the sides of the pillar adjacent to the tether (as illustrated). The goat walks once around the pillar in the direction shown, keeping the rope always taut. The goat's path is shown.



- How far does the goat walk?
- How far would the goat walk if the pillar's cross-section was a square of side 3 metres?
- How far would the goat walk if the pillar had a circular cross-section of circumference 12 metres? (Assume the rope forms a tangent to the circle initially.)

(10 marks)

14. The diagram shows an adapted snooker table which has length  $m$  and width  $n$  where  $m$  and  $n$  can only take integer (whole-number) values. The table only has pockets at the four corners A, B, C, and D.



The ball is started from point A and is directed at  $45^\circ$  as shown in the diagram. When the ball hits any cushion it rebounds symmetrically at  $45^\circ$  as shown in the diagram. Assume that the table is smooth and so the ball continues until it enters one of the pockets. (Ignore any complicated effects of the ball hitting the jaws of the pockets.)

- i) Which pocket does the ball end up in if:
  - a.  $m = 3$  and  $n = 2$
  - b.  $m = 5$  and  $n = 3$
- ii) If the ball enters pocket C, what can you say about  $m$  and  $n$ ?
- iii) If the ball enters pocket D, what can you say about  $m$  and  $n$ ?

(10 marks)

15. In a football league there are 10 teams and, over a season, each team plays every other team twice, with 3 points for a win and 1 point for a draw. At the end of one season, exactly one third of the games played were draws. Every team finished with a different number of points.

- The team that finished first had beaten every other team in the league at least once, and the team that finished second had also beaten every other team in the league at least once.
- The bottom two teams had been beaten twice by both the top two teams.
- The bottom team finished with 18 points.

By how many points was the second placed team ahead of the fourth placed team?

(10 marks)