

The
ARNOLD HAGGER
Mathematics Prize Exam

Wednesday 27th January 1999

7-15 p.m.

1½ hours

Calculators are NOT allowed

Answer as many questions as you can.

All solutions should be fully explained.

*Good answers to a small number of questions will gain greater credit
than a large number of fragmentary answers.*

The marks for each question are shown in brackets at the end of the question [].

1 How many digits are there in $2^{12} \times 5^8$? [3]

2 Each of a group of 50 girls is either blonde or brunette and is either blue or brown-eyed.

There are 14 blue-eyed blondes, 31 brunettes and 18 brown-eyed girls. How many brown-eyed brunettes are there? [5]

3 What is the number of points which are equidistant from a circle and two parallel tangents to that circle?

Illustrate your answer with a sketch. [3]

4 Two classes took the same test. One class of 20 students had an average mark of 80% and the other class of 30 students had an average mark of 70%.

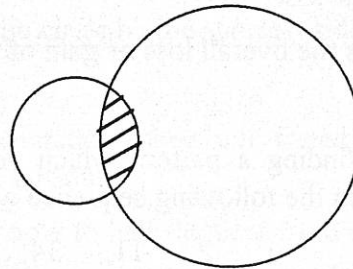
What was the average mark of all the students? [3]

5 If the radius of a circle is a rational number, is the area (a) rational, (b) irrational, (c) integral or (d) a perfect square? [2]

6 If the radius of a circle is increased by 100%, by what percentage is the area increased? [3]

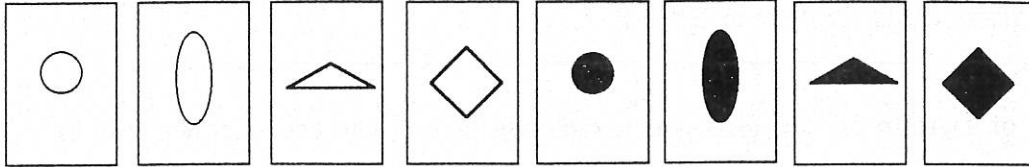
7 Two overlapping circles have radii 1 cm and 3 cm. [3]

If the area of the shaded region is $\frac{\pi}{2} \text{ cm}^2$, what is the total area of the figure?



8 Prove that if all the multiples of 3 from 9 to 340 were multiplied together the resulting product would have 22 zeros. [6]

- 9 Your friend has a pack of eight cards and invites you to choose one at random.



He then asks you three questions to which you answer 'yes' or 'no', after which he identifies the card chosen. It later turns out that he can always do this by asking the same three questions, in the same order. What might the questions be? [4]

- 10 A palindromic number is one which has the same value when the order of its digits is reversed.

How many palindromic numbers are there between 10 and 1000? [4]

- 11 The base of a rectangle is increased by 10% but the area is unchanged. By what percentage has the width been decreased? [4]

- 12 A rectangular enclosure is half as wide as it is long and is completely enclosed by x metres of fencing. What is the area of the enclosure in terms of x ? [4]

- 13 A house and a bungalow were sold for £120 000 each. The house was sold at a loss of 20% of its original cost and the bungalow was sold at a profit of 20% of its original cost.

What was the overall loss or gain of the transaction? [5]

- 14 By first finding a pattern which you should demonstrate, find the next two numbers in the following sequence which follow the pattern you have noticed:

1, 2, 11, 34, 77, 146, ..., ... [5]

- 15 A pandigital contains each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 just once.

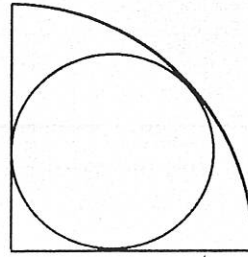
Find a pandigital which is divisible by 720. [5]

- 16 The sums of money in the hands of three card players A , B , and C are in the ratios 7 : 6 : 5 at the beginning and 6 : 5 : 4 at the end of a game.

Who won and who lost and by how much? [5]

- 17 Simplify $\frac{a^2 - b^2}{ab} - \frac{ab - b^2}{ab - a^2}$ [5]
- 18 Prove that the sum of any eight consecutive odd numbers is divisible by 16. [6]
- 19 A telephone pole is 60 ft. high, $1\frac{1}{2}$ ft. diameter at the base, has a circular cross section throughout and tapers uniformly to the top where the diameter is 1 ft. What is its volume? [6]
- 20 How many perfect squares can there be of the form $1! + 2! + 3! + 4! + 5! + \dots$? [6]
- 21 What is the greatest number of Friday 13ths that can occur in one year? [6]
- 22 Here is a little arithmetical puzzle that you should all be able to solve!
The head of a fish was 9 cm. long, the body was as long as the head and the tail together, and the tail was as long as the head and half the body.
How long was that fish? [6]
- 23 Show that $a^2 + b^2$ can never be three more than a multiple of four. [6]
- 24 A circle has an inscribed triangle whose sides are $5\sqrt{3}$, $10\sqrt{3}$ and 15. What is the value of the angle subtended at the centre of the circle by the shortest side? [7]
- 25 "I say, Rackbrane, what is the time?" an acquaintance asked our friend the professor the other day. The answer was certainly curious.
"If you add one quarter of the time from noon till now to half the time from now till noon tomorrow, you will get the time exactly."
What was the time of day when the professor spoke? [7]
- 26 The difference of the squares of the positive integers a and b is 924. Find all possible values of a and b . [10]

- 27 What is the ratio of the area of the inner circle to the area of the quadrant?



[8]

- 28 If $x + y = 1$ and $x^2 + y^2 = 221$, determine the values of $x^3 - y^3$.

[10]

- 29 The equation $\frac{x^2 - bx}{ax - c} = \frac{m - 1}{m + 1}$

has roots which are numerically equal but of opposite signs.

Find m in terms of a , b and c .

[8]

TOTAL [155]