A
R R
N N N
O O O O
L L L L L
D D D D D D

H
A A
G G G
G G G G
E E E E E
R R R R R R

MATHS PRIZE EXAM

1995

22nd FEBRUARY

7-15 p.m.

1½ hours

Answer as many questions as you can.

All solutions should be fully explained.

Good answers to a small number of questions will gain greater credit than a large number of fragmentary answers.

The marks for each question are shown in brackets at the end of the question [ ].

1.

A

R R

N N N

O O O O

L L L L L

D D D D D D

H H H H H H H

A A A A A A A A

G G G G G G G G G

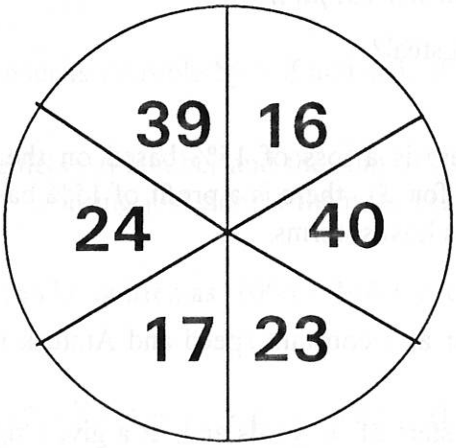
G G G G G G G G G G

E E E E E E E E E E E

R R R R R R R R R R R R

Beginning with the letter A at the top of the triangle and reading down, always passing from a letter to a diagonally adjacent letter, in how many ways is it possible to read **ARNOLD HAGGER** (omitting the space between the words)? [5]

2.



How many darts, in which areas, are needed to score exactly one hundred on this dartboard? [5]

3. In a sports club of 120 members 80 play golf, 50 play tennis and 60 play squash: 22 play squash and tennis, 28 play tennis and golf, 44 play squash and golf and 10 play all three. How many non-playing members are there in the club? [5]

4. Find the real value of x such that $\frac{64^{x-1}}{4^{x-1}} = 256^{x}$. [5]

5. If all the calculations in

$$S = 1! + 2! + 3! + 4! + \ldots + 99!$$

are performed correctly, find the units digit in the value of S.

(Note: $n! = n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 3 \times 2 \times 1$) [5]

6. The area between two concentric circles is $12\frac{1}{4}\pi$ square centimetres. Find the length of a chord of the larger circle which is a tangent to the smaller circle. [8]

7. *The cellarer, John, was taken before the Abbott accused of stealing wine. The Abbott looked grave and asked John why he should not be punished.*

John said: "My Lord Abbott, I have taken a pint from the cask every day this month, it now being the thirtieth, and if my Lord Abbott can tell me exactly how much good wine I have taken let him punish me as he will."

"Why, knave" said the Abbott, "that is thirty pints".

"Nay, nay," replied John, "for each day that I stole a pint of wine I put a pint of water in its place, there being one hundred pints in the cask at the start."

John suffered no punishment for his sad fault!

How much good wine did John steal? [8]

8. If an item is sold for £x, there is a loss of 15% based on the cost price. If, however, the same item is sold for £y, there is a profit of 15% based on the cost price. Find the ratio $y : x$ in its lowest terms. [8]

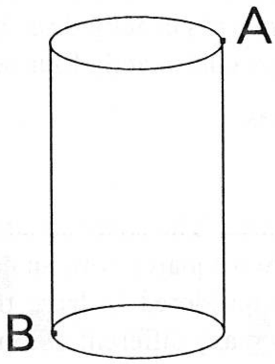
9. Mr. Hall drives his Land-Rover at a constant speed and Ataturk runs y times as fast ($y > 1$).

Ataturk gives Mr. Hall a head start of x yards and, at a given signal, they both start off in the same direction and continue to move in a straight line. Find the number of yards Ataturk must run to catch up Mr. Hall. [8]

10. If 4 men and 1 dog can dig 12 holes in 4 days and if 2 men and 1 dog can dig 4 holes in 2 days, how long will it take 1 man and two dogs to dig 15 holes? [8]

11. In how many ways can the Jones family (Mr. and Mrs. Jones, Ann and Barry) and the Robinson family (Mr. and Mrs. Robinson, Edward and Fiona) be arranged around a circular dining table, in such a way that no two children of the same family occupy adjacent seats and Mr. Jones occupies the seat nearest the door? [10]

12.



A spider starts at A and walks two and a half times round the cylinder to reach B. If the radius of the cylinder is 3 and the height is 20π, find *exactly* the shortest distance the spider could have walked. [10]

13. Let $x = \sqrt{11} + \sqrt{2}$. Show that $x^2 - 2\sqrt{2}\,x$ is a positive integer.

Hence, or otherwise, prove that x is irrational.

(You may assume that $\sqrt{2}$ is irrational). [10]

14. Prove that a whole number is divisible by 9 if and only if the sum of its digits is divisible by 9.

X is any positive whole decimal number and the sum of its digits is Y. If the sum of the digits of X - Y, omitting just one of them, is 61, determine the value of the other digit.

(*Hint:* the number *abc* can be written as $100a + 10b + c$, etc.) [10]

15. You are given that $1 + 2 + 3 + 4 + \ldots + n = \frac{1}{2}n(n+1)$.

In the equation

$$N = n_1 + n_2,$$

n_1, n_2 and N are non-negative integers and N is fixed. Show that there are $N + 1$ possible solutions for n_1 and n_2.

In the equation

$$N = n_1 + n_2 + n_3,$$

n_1, n_2, n_3 and N are non-negative integers and N is fixed. How many solutions are there for n_1, n_2 and n_3? [15]

16. A penny is rolled onto a large flat, horizontal grid of squares.

The side of the square is $3 \times$ the radius of the penny.

Find (a) the probability that the penny lies entirely within a square,
(b) the probability that the penny cuts exactly one boundary,
(c) the probability that the penny cuts exactly two boundaries,
(d) the probability that the penny cuts exactly three boundaries,
(e) the probability that the penny cuts exactly four boundaries.

A boundary is a line between two squares. [15]

17. A 3×3 floor tile comprises 9 unit squares. The small squares are to be coloured Red, White or Blue in such a way that two squares with an edge in common must be of different colours. Two tiles are considered to have the same colouring if one can be rotated onto the other. How many different coloured floor tiles can be produced.

(*Hint:* consider how many ways there are to colour the cross obtained by deleting the four corner squares.) [15]