

**SHREWSBURY SCHOOL**  
**THE ARNOLD HAGGER MATHEMATICS PRIZE**

Wednesday 17th February 1993

Time allowed 1 hour 30 minutes

You may attempt as many questions as you like, in any order. Be careful to make your methods clear by including all working and reasoning.

Calculators may **not** be used.

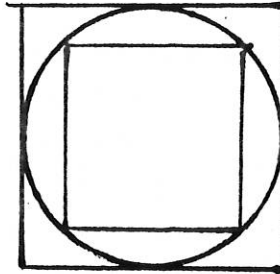
1. A positive whole number consists of two digits differing by 1. If the square of the number formed by reversing the digits is added to the square of the original number, the sum is 585. There is only one such pair of numbers. Find it.
2. This is the menu at a roadside cafe. Work out the cost of each individual item. (Sausage, Egg, Bacon, Chips )

MENU

Sausage, egg & chips	2.20
Bacon, egg & chips	2.29
Sausage, bacon & chips	2.34
Sausage, bacon & egg	2.08

3. 100 digits are chosen at random. What is the probability that there will be at least one pair of numbers,  $a$  and  $b$ , such that each digit is used exactly once in forming the two numbers, and such that  $a^2 = b$ . No number may begin with the digit 0.
4. The cold water tap of a bath runs one third as fast again as the hot water tap. The hot tap fills the bath in 14 minutes. How long will it take to fill the bath with both taps running?
5. In a group of 100 boys, 71 have had measles, 61 have had mumps and 40 have had chickenpox.
  - a) What is the largest number who could have had all the illnesses?
  - b) How many could have had none of these illnesses?
  - c) If only one boy has had all three different illnesses and no boy has escaped all three, how many have had exactly two of the three illnesses?

6. If I draw a circle to fit exactly in a square, and then a square to fit into the circle I get this picture. What fraction of the large square is the small square?



7. Smith, Jones and Robinson are the engineer, brakeman and fireman on a train, but not necessarily in that order. Riding the train are three passengers with the same three surnames, to be identified in the following premises by a 'Mr' before their names.

Mr. Robinson lives in Los Angeles.

The brakeman lives in Omaha.

Mr. Jones long forgot all the algebra he learnt at school.

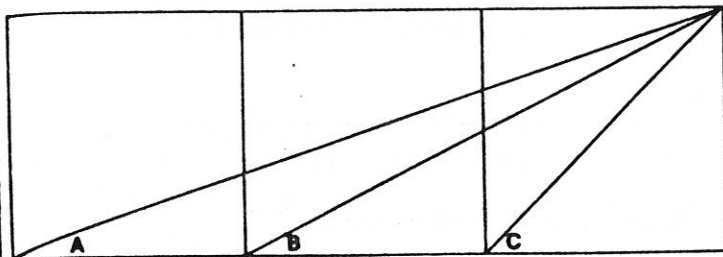
The passenger whose name is the same as the brakeman's lives in Chicago.

The brakeman and one of the passengers, a distinguished mathematical physicist, attend the same church.

Smith beat the fireman at billiards.

Who is the engineer?

8. Using only elementary geometry (not even trigonometry) prove that angle C in the figure below equals the sum of angles A and B.



*Prove that angle A plus angle B equals angle C.*

9. A cylindrical glass is 4 inches high and 6 inches in circumference. On the inside, 1 inch from the top, is a drop of honey. On the outside, 1 inch from the bottom and directly opposite, is a fly. What is the shortest path by which the fly can walk to the honey, and exactly how far does the fly walk?

10. The owner of a pet shop bought a certain number of hamsters and half that many pairs of parakeets. He paid £2 each for the hamsters and £1 each for the parakeets. On every pet he fixed a retail price that was 10% more than what he paid for it.

After all but seven of the pets had been sold the owner found that he had taken in for them an amount of money exactly equal to what he had paid for all of them. His potential profit, therefore, was the combined retail value of the seven remaining animals. What was this value?

11. If a circle is inscribed in a square and between it and the 4 vertices 4 other circles are drawn and so on continually, show that the ratio of the sum of the perimeters of all these circles to that of the first circle is:

$$2\sqrt{2} - 2 : 1$$

Find the ratio for the corresponding areas.

12. a) A man standing on a platform notes that a train going in one direction takes 3 seconds to pass him, a train of the same length in the other direction 4 seconds. How long did they take to pass one another?
- b) Of a swarm of bees on a tree, the square root of half their number flew away. Eight ninths of the original number then departed, leaving just two behind. How many were there at first?