

SHREWSBURY SCHOOL

THE ARNOLD HAGGER MATHEMATICS PRIZE

Wednesday 17th February 1988

Time allowed:  $1\frac{1}{2}$  hours.

You may attempt as many questions as you like,  
in any order. Show working and/or reasoning -  
answers alone are not sufficient.

You may use calculators and tables.

1. The digits of 1988 and the processes of +, -,  $\times$ ,  $\div$ , and raising to a power may be used generate numbers. Each digit may be used once only.

For example;  $1 + 9 + 8 + 8 = 26$

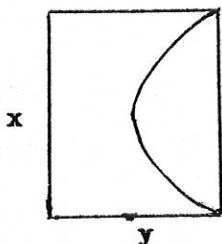
$$8 + 8 - 9 - 1 = 6$$

$$918^8 = 5.04 \times 10^{23}$$

What is the largest number you can make? How many digits does this number have ?

2. The semicircle divides the rectangle into two parts of equal area.

Find the ratio of the length to the width of the rectangle ( i.e.  $x:y$  )



3. What is the remainder when  $10^{21}$  is divided by 764 ?  
( There are better ways of doing this than using long division! )
4. The diameter of a roll of tape is 12 cm when unused and 10 cm when all used up . What is its diameter when it is half used up ? ( Answer to 2 decimal places ).
5. Are there integers a and b such that  $5a^2 - 7b^2 = 9$  ?  
Either find them or show that they do not exist.
6. Consider right angled triangles for which the lengths of all sides are integers. Find all such triangles which have their areas numerically equal to their perimeter . Your answer should include an explanation of why your list is complete.

7. i) A square with sides of length 'n' units has one diagonal drawn across it from one corner to the opposite one. The square is divided into  $n^2$  unit squares. How many of these unit squares do not have a diagonal drawn on them? Use this to find a formula for :

$$1 + 2 + 3 + \dots + n$$

- ii) n lines are drawn in the plane so that no pair is parallel, and no three meet in a single point. Find a) the number of regions into which the plane is divided.  
b) the number of these regions which do not extend to infinity.

8. A cylinder has diameter 6 cm and height 8 cm. This cylinder is tilted so that the bottom makes an angle of  $45^\circ$  with the horizontal and then partially filled with water as shown in the sketch. How much water is in the cylinder?

