SHREWSBURY SCHOOL

MATHEMATICS PRIZE EXAMINATION

Wednesday 20th February 1985

7-15 p.m. to 8-45 p.m.

Answer as many questions as you like, in any order.

All answers must be fully explained.

[TURN OVER

SECTION A

1. P is the increase in the circumference of a circle resulting from an increase of π units in the diameter. Find P.

(3 marks)

2. Find the real value of x such that 64^{x-1} divided by 4^{x-1} equals 256^{x}.

(3 marks)

3. A positive number is mistakenly divided by 6 instead of being multiplied by 6. Based on the correct answer find the percentage error thus committed.

(3 marks)

4. If an item is sold for £x, there is a loss of 15% based on the cost price. If, however, the same item is sold for £y, there is a profit of 15% on the cost price. Find the ratio y:x.

(3 marks)

5. The area between two concentric circles is $12\frac{1}{2}\pi$ square inches. Find the length of the chord of the larger circle which is a tangent to the smaller circle.

(3 marks)

SECTION B

6. A circle passes through the vertices of a triangle with side lengths $7\frac{1}{2}$, 10 and $12\frac{1}{2}$. Find the radius of the circle.

(4 marks)

7. Find the number of ways that £10 can be changed into 50 pence and 10 pence pieces, with at least one of each type of coin being used.

(4 marks)

8. How many points are there in common to the graphs of $(x-y+2)(3x+y-4) = 0$ and $(x-y+2)(2x-5y+7) = 0$.

(4 marks)

9. Side AB of a triangle has length 8 inches. Line DEF is drawn parallel to AB so that D is on segment AC and E is on segment BC. Line AE extended bisects the angle FEC. If DE has length 5 inches, find the length of CE.

(4 marks)

10. Let P equal the product of 3,659,893,456,789,325,678 and 342,973,489,379,256. Find, without the use of logarithms, the number of digits in P.

(4 marks)

11. If all the calculations in

$$S = 1! + 2! + 3! + \ldots + 99!$$

are performed correctly, find the units' digit in the value of S.
(N.B. $N! = N.(N-1).(N-2). \ldots 3.2.1$.) (4 marks)

SECTION C

12. How many prime numbers are there greater than $N!$ and less than $N! + N$? (Be careful!)
(N.B. $N! = N.(N-1).(N-2). \ldots 3.2.1$.) (5 marks)

13. If $x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}$, t positive and not equal to 1, find an equation connecting x and y but not involving t.

(5 marks)

[TURN OVER

14. A painting 18" x 24" is to be placed into a wooden frame with the longer dimension vertical. The wood at the top and bottom is twice as wide as the wood on the sides. If the frame area equals that of the painting itself, find the ratio of the smaller to the larger dimension of the framed painting.

(5 marks)

15. Ataturk runs with constant speed and Florence runs y times as fast (y greater than 1). Florence gives Ataturk a head start of x yards, and, at a given signal, they both start off in the same direction. Find the number of yards Florence must run to catch Ataturk.

(5 marks)

16. ABC is a triangle and squares are described outwards on the sides AB and AC. The circles which circumscribe these squares meet at P and at A. Find the locus of P when B and C are fixed and A varies

(5 marks)

17. A bookseller bought from a wholesaler twenty pamphlets at a certain price, and a few days later ten other pamphlets paying ten new pence less for each pamphlet. He noticed that the two accounts were expressed by the same two digits in pounds but reversed in order. Find the price of the more expensive pamphlet.

(5 marks)

18. ABCD is a square described outwards on the hypoteneuse AB of a right angled triangle OAB. The bisectors of the angles OAB and OBA meet at I. AC and BD meet at K. Prove that O, I and K lie on a straight line.

(5 marks)