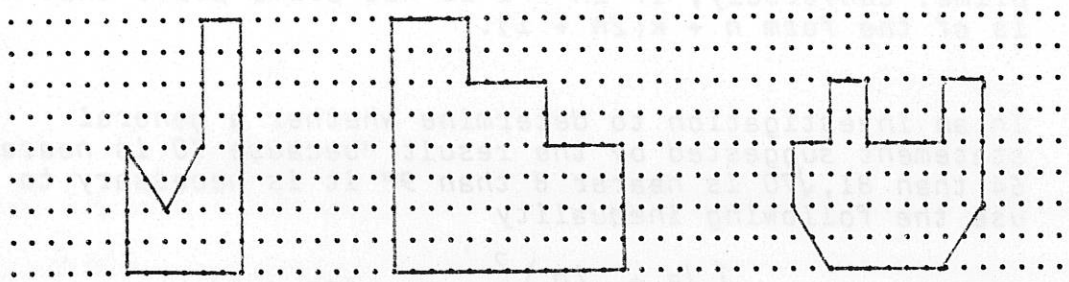
SHREWSBURY SCHOOL 1983 MATHEMATICS PRIZE

1½ HOURS

You may attempt as many questions as you wish. Complete answers to a few questions are preferred to a large number of fragmentary solutions.

1.



(a) (b) (c)

The dots in the above diagram form a square lattice. By the addition of a line (not necessarily straight) divide each shape into two ~~two~~ congruent pieces.

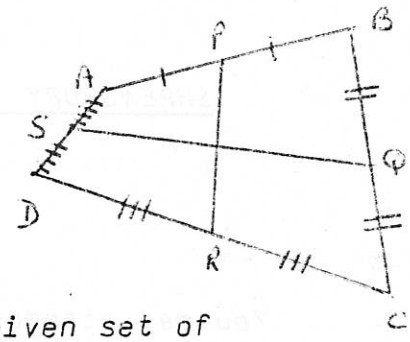
2. (i) Observations of the paving of Pride Hill show that three men can pave a certain area in five days. A different group of five men are observed paving a similar sized area in four days. If all work together, at the same rate as before, how long will it take to pave such an area ?
(ii) Repeat the above problem to find the time, T, taken for a+b men to do the job when a men are observed to do the work in T_1 days and b men in T_2 days. (A familiar formula?)

3. If X is an irrational number prove that 1 + X and 1/X are also irrational numbers.

4. a and b are two numbers chosen such that their difference is even. Show that the product of the two numbers can be written as the difference of two squares. Why is the result not true if the difference is odd ?

5. When a certain piece of paper is folded into two its shape is found to be similar to the original shape. Determine the ratio of the length to breadth. This is the principle upon which the metric sizes of paper are based.

6. ABCD is a quadrilateral with no sides of equal length or parallel. The points P,Q,R and S are the mid points of sides AB,BC,CD and DA respectively. Prove that the lines PR and QS bisect each other.



7. Draw a flow diagram which will input a given set of numbers and print them out in ascending order.

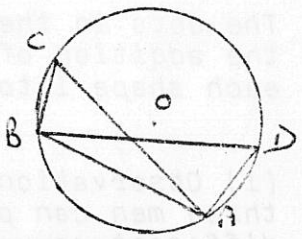
8. N is a number chosen such that $N = n + k(2n + 1)$ where n and k are integers. Prove that the number $2N + 1$ is not prime. Conversely, if $2N + 1$ is not prime prove that N is of the form $n + k(2n + 1)$.

9. In an investigation to determine whether a general statement suggested by the result "because 70 is nearer 64 than 81, $\sqrt{70}$ is nearer 8 than 9" it is necessary to use the following inequality

$$\left(\frac{\sqrt{a} + \sqrt{b}}{2}\right)^2 < \tfrac{1}{2}(a + b)$$

Prove that, provided $a \neq b$, the above inequality is true.

10. A,B,C,D are 4 points on the circumference of a circle centre O. Prove that the angles ACB and ADB are equal.



11. r indistinguishable red balls are to be placed into n boxes. Determine in how many different ways this can be done. No marks will be awarded for writing down the correct answer only.

12. W is the set of all 2 x 2 matrices of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a,b,c,d are real numbers and $ad - bc = 2^n$ for some integer n, positive, negative or zero. (The integer n is not fixed and may be different for different elements of W.)
(a) Prove that W is closed under matrix multiplication i.e. if $X, Y \in W$ then $XY \in W$ for all matrices X and Y.
(b) Does every matrix of W have a multiplicative inverse in W? (Either prove that this is true or give a counter-example.)

A.W.H.
February 1983