SHREWSBURY SCHOOL   3 Feb 1982 1½ hrs FDH

MATHEMATICS PRIZE 1982

You may answer as many questions as you can, in any order. All questions carry the same number of marks, but more credit will be given for complete answers than for a large number of fragments. Take care to explain your reasoning carefully - little credit will be given for mere answers. Calculators and mathematical tables are allowed.

1. If the digits of any three-digit number are rearranged, in any order, to form another number of three digits, prove that the difference between the two numbers is always divisible by 9. Explain why the same result is true for numbers with <u>any</u> number of digits.

2. A and B are two airfields, B being due East of A. Asimple qualifyung test for new pilots is to fly from A ao B and back again. Prove that the time taken for the test is alwys increased if there is a wind blowing in an East-West direction. (i.e. is always greater than if the air is still.) Discuss briefly (but as precisely as possible) what would happen if the pilot set out from A not realising that there was a wind blowing from the North.

3. It takes x minutes for y greedy boys to eat z Freddos.
   (i) How long does it take x boys as greedy as before to eat y Freddos?
   (ii) How many Freddos do z boys as greedy as before eat in y minutes?
   (iii) If the heap of Freddos diminishes at a faster rate in (i) than in (ii), what must be true about x,y,z?

4. (i) If p is a prime, greater than 3, prove that $p+2$ and $p+4$ cannot both be primes.
   (ii) Find a prime q, greater than 3, such that neither $q+6$ nor $q+12$ are primes.
   (iii) If p is a prime, greater than 3, then can $p+4$ and $p+8$ both be primes? Explain your answer.
   (iv) What can you say briefly about the possibility or impossibility of finding a prime r such that $r+33000$ and $r+66000$ are both primes?

5. Two lines l and m meet at O. The point A is on l and B is on m such that OA = AB. If the angle AOB = θ, where θ lies between 0 and 90 degrees, draw the diagram for an angle θ is approximately equal to 20 degrees, adding points C on l and D on m such that BC = AB and CD is parallel to AB.
   As θ increases, the diagram changes: draw further diagra,s to illustrate this and indicate the two values of θ at whcih the changes occur.
   For the three different versions of the diagram, dind three separate expressions for the size of the angle BCD in terms of θ.

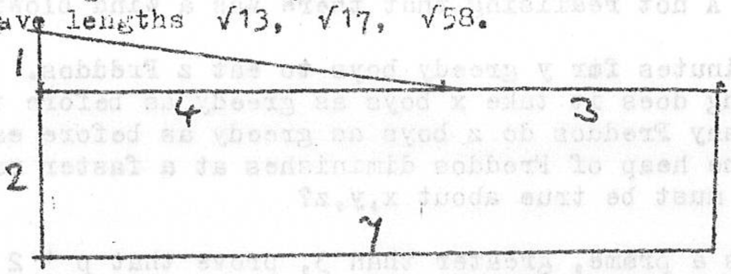
6. Cycling at a steady speed from his home to school, Fred was overtaken by a bus which left his home b minutes after he left and arrived at his school c minutes before him. If the bus travels at a steady speed, how far from home was Fred when the bus overtook him if it is d km from his home to school?
   If the bus travels at f km/hr, how fast does Fred cycle?

7. (a) Two altetudes of a triangle are 4 cms and 5 cms in length. By considering the area of the triangle and using the fact that the sum of the lengths of two sides of a triangle is always greater than the length of the third side, find the greatest and least possible values of the length of the third altitude of the triangle.

   (b) Two circles have the same centre and their radii are 10 cms and 13 cms. Calculate the length of the longest rectangle of width 2 cms which can be cut out from the area between the circles. How many such rectangles could be cut from the whole of this area?

8. Your school, called A for convenience, is to be host to five other schools, namely B,C,D,E and F, for a cricket festival at the end of term. Each school is to play each of the others once, the festival lasting for five days, your school having of course three excellent pitches available. Each match is a one-day match. You have to arrange which school plays which other school each day and on which pitch. You start by arranging Day 1: A v B on pitch 1, C v D on pitch 2 and E v F on pitch 3. You also arrange your own school's programme, namely A v B on day 1, A v C on day 2, A v D on day 3, A v E on day 4 and A v F on day 5. You also arrange for B v E on day 2, and to make things more interesting, no school plays on the same pitch on two successive days. Draw up a complete schedule of your arrangements for the festival and, in particular, on what day and on what pitch does B play against C?

9.(a) With the help of the diagram (which is a sketch of a rectangle and a right angles triangle with lengths as shown) calculate the area of a triangle whose sides have lengths $\sqrt{13}$, $\sqrt{17}$, $\sqrt{58}$.



9(b) A triangle has sides 20cm, 25 cm, 32 cm. Explain clearly whether the area is less than equal to or greater than 250 cm². An attempted solution by measurement is not acceptable in this question.

10.

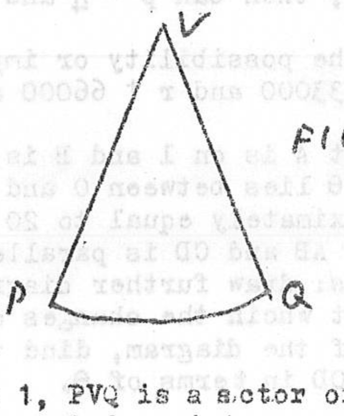


FIG 1

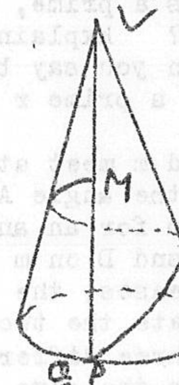


FIG 2

In figure 1, PVQ is a sector of a circle S and $P\hat{V}Q < 180°$. Figure 2 shows PVQ curled up into a cone (with horizontal circular base) by laying VP and VQ along each other. M is the mid-point of VP. A certain path is shown spiralling once round on the surface of the cone from P to M. It is the shortest such path.

(i) Explain why, if in figure 1 $P\hat{V}Q$ is large enough, the path in figure 2, though ascending from P, descends to reach M.

(ii) If in figure 1, PV = 10 cm and $P\hat{V}Q = 90°$, find by calculation, leaving square root signs in, the shortest distance from V to a point of the path in figure 2.

(iii) Find with brief explanation the angle size, θ°, such that if in figure 1 $P\hat{V}Q < θ°$, then the path ascends all the way to reach M, but if in figure 1 $P\hat{V}Q > θ$, then the path eventually descends to reach M.