# MATHEMATICS PRIZE 1970.

SDB

1. (i) If $a + b + c + d = 0$ ptove that

$$(a + b)^3 + (a + c)^3 + (a + d)^3 + (b + c)^3 + (b + d)^3 + (c + d)^3 = 0.$$

(ii) If $x^2 = x + 5$ prove that $x^5 = 41x + 55$.

2. A sequence of triples $(a_i, b_i, c_i)$ for $i = 1,2,3,\ldots$ is formed as follows :

$a_1 = 3,\quad a_{i+1} = a_i + 2 \quad \Rightarrow \quad a_1 = 3,\ a_2 = 5,\ a_3 = 7$ etc

$b_1 = 4,\quad b_{i+1} = b_i + 4(i+1) \quad \Rightarrow \quad b_1 = 4,\ b_2 = 12,\ b_3 = 24$ etc.

$c_1 = 5,\quad c_{i+1} = c_i + 4(i+1) \quad \Rightarrow \quad c_1 = 5,\ c_2 = 13,\ c_3 = 25$ etc.

Show that $a_i^2 + b_i^2 = c_i^2$ for all positive integers i.

3. Let H be a subgroup of a group G. Define the relation R on the set G by

$$x R y \iff x * y^{-1} \in H \quad \text{for } x, y \text{ in } G.$$

Show that the relation R is an eqivalence relation.

4. ABC is a tfiangle and squares are described outwards on the sides AB and AC. The circles which circumscribe these squares meet at P and at A. Find the locus of P when B and C ar fixed and A varies.

5. Show that if x and y are any two rational numbers, such that $x < y$, then there is

(i) a rational number a such that $x < a < y$;

(ii) an irrational number b such that $x < b < y$.

6. Find the missing digits in the following multiplication;

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      4 * *
        3 *
  ---------
    3 6 * *
  * * 7 *
  ---------
  * * 3 * *
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7. ABCD is a square described outwards on the hypoteneuse AB of a right angled triangle OAB. The bisectors of the angles OAB and OBA meet at I. AC and BD meet at K. Prove that O, I, K lie on a straight line.

P.T.O.

8. A set of dominoes, each showing two numbers, can be arranged thus :

(0, 0)
(0, 1), (1, 1)
(0, 2), (1, 2), (2, 2)
(0, 3), (1, 3), (2, 3), (3, 3)
etcetera

giving 28 dominoes in a set from 'double-blank' to 'double-six'.

In a set from 'double-blank' to 'double-p',

(i) how many dominoes are there,
(ii) find the number of times the number q (less than p) appears,
(iii) the 'value' of a domino being the sum of the two numbers appearing on it, find the sum of all the values.
(iv) show that the average value of a domino is p.