

SHREWSBURY SCHOOL

MATHEMATICS PRIZE, 1963

1. A barn, 42 ft. square, stands in a field. A goat is tied to the mid point of one wall of the barn by a rope 105 ft. long. Find the area of the field that the goat can graze ($\pi=22/7$).

2. What is the largest number which leaves the same remainder on division into each of the four numbers 701, 1059, 1417 and 2312 ?

3. The circles APQ, AHK touch externally at A. If PAH is a straight line and if the chords QP, HK meet, when produced, at a point T on the tangent at A, such that AT bisects the angle QTK, prove that QA=AK.

4. (i) Show that if $a(a+b)+b(b+c)+c(c+a)=0$, and a, b and c are real, then $a=b=c=0$.

(ii) If $ABC=AB+BC+CA$, what is CAB ?

(The number ABC means that A is the hundred digit, B is the ten digit and C the unit digit. A, B and C are all different.)

5. Four homing missiles, A, B, C and D, are launched from the vertices of a square of side 30 miles. A homes on B, B on C, C on D, and D on A. If the missiles travel at one mile a second, find how long after launching they will collide.

6. A bookseller bought from a publisher 20 books at a certain price, and a few days later 10 other books paying 2s. less for each book. He noticed that the two accounts were expressed by the same two digits in pounds, but of course reversed in order. What was the price of the more expensive book ?

7. ABCD is a quadrilateral in which AB, DC are parallel and $AB+DC=BC$. Prove that the bisectors of the angles B, C meet at right angles at the mid-point of AD.

8. A small rhombicosidodecahedron is a symmetrical solid body with plane faces, each of which is a regular polygon. Four faces meet at each vertex—a square, a pentagon, another square and a triangle. The two squares which meet at each vertex are separated by a pentagon on one side and a triangle on the other, *i.e.* each pentagon is surrounded by 5 squares, each triangle by 3 squares, and each square by 2 pentagons and 2 triangles.

How many vertices and how many faces has a small rhombicosidodecahedron ?