

SHREWSBURY SCHOOL

MATHEMATICS PRIZE, 1961

1. Find the smallest whole number which leaves remainders 2, 4, 6, 10 when it is divided by 3, 5, 7, 11 respectively.

2. Solve:

$$\begin{aligned} b+c+d &= 19, & a+c+d &= 16, \\ a+b+d &= 14, & a+b+c &= 11. \end{aligned}$$

3. A circular piece of paper is folded about a chord AB, and a circle, centre A, cuts the arcs in P and Q. Prove that B, P, Q are in a straight line.

4. Find the law governing the following two series of numbers, and write down the next term in each series :—

(i) 0, 1, 2, 9, 44, 265,

(ii) $\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \dots$

5. ABCD is a parallelogram and P is a point inside such that the angles APB and CPD are supplementary. Prove that the angles PAD, PCD are equal.

6. A vessel contains 10 gallons of milk. When a gallon of its contents is drawn out, a gallon of water is poured in. After how many such operations will the contents of the vessel contain less than 10 per cent of milk?

7. Prove that the sum of the diagonals of a quadrilateral is less than its perimeter but greater than half its perimeter.

8. One side of a triangle is one-third of the perimeter, and the area of the triangle is three-fifths that of the equilateral triangle described on that side. Prove that the sides of the triangle are in the ratios 3 : 5 : 7.